

HOMEWORK 8  
Math 104 - Dr. Evans  
UCSD Winter 2004  
Due Thursday, March 11, 5:00pm

1. Is some positive power of  $2 + \sqrt{11}$  congruent to 1 modulo 7? Justify your answer.
2. If  $N(a + b\sqrt{n}) = a^2 - nb^2$  is relatively prime to the odd prime  $q$ , show that some positive power of  $a + b\sqrt{n}$  is congruent to 1 modulo  $q$ .
3. Let  $a, b, K$ , and  $M$  be positive integers and  $q$  be an odd prime. If  $a + b\sqrt{m}$  has order  $M$  (mod  $q$ ) and  $(a + b\sqrt{m})^K \equiv 1 \pmod{q}$ , show that  $M|K$ . (Recall that the *order*  $M$  is the power of  $a + b\sqrt{m}$  such that  $(a + b\sqrt{m})^M \equiv 1 \pmod{q}$  and  $(a + b\sqrt{m})^n \not\equiv 1 \pmod{q}$  for  $1 \leq n \leq M - 1$ .)
4. Prove that if  $q|(a + b\sqrt{m})$ , then  $q|(a - b\sqrt{m})$ .