Fibonacci and Lucas Numbers

Mathematical Explorations – Math 110

Block 2, Fall 2007

1. How many ways can you tile a $1 \times n$ board with 1×1 and 1×2 dominoes? Try it for $n = 1, 2, 3, 4, 5$ and see if there's a pattern. For fun, try to find the number of ways to tile a $2 \times n$ board with only 1×2 dominoes.

2. Find a formula for adding the first n Fibonacci numbers together. Try the same for adding the *squares* of the first n Fibonacci numbers together.

3. By experimenting the first ten or so cases, fill in the blanks with the appropriate expression of Fibonacci and/or Lucas numbers. (Hint: For the third and fourth, also look at the squares of Fibonacci and Lucas numbers.)

$F_{n+1} + F_{n-1} =$ _____________ $L_{n+1} + L_{n-1} =$ _____________

$F_{n+1} \cdot F_{n-1} =$ _____________ $L_{n+1} \cdot L_{n-1} =$ _____________

$L_n^2 - 5F_n^2 =$ _____________ $L_n^2 - L_{2n} =$ _____________

Challenge Problem

Prove that that F_n divides F_{kn} for all $k = 1, 2, 3, 4, \ldots$ You may want to use the formula $F_{m+n} = F_{m+1}F_n + F_mF_{n-1}$.

Fibonacci and Lucas Numbers

$$F_{n+2} = F_{n+1} + F_n \qquad L_{n+2} = L_{n+1} + L_n$$

n	1	2	3	4	5	6	7	8	9	10	11	12
F_n	1	1	2	3	5	8	13	21	34	55	89	144
L_n	1	3	4	7	11	18	29	47	76	123	199	322

n	13	14	15	16	17	18	19	20
F_n	233	377	610	987	1597	2584	4181	6765
L_n	521	843	1364	2207	3571	5778	9349	15127

n	21	22	23	24	25	26
F_n	10946	17711	28657	46368	75025	121393
L_n	24476	39603	64079	103682	167761	271443

n	27	28	29	30	31	32
F_n	196418	317811	514229	832040	1346269	2178309
L_n	439204	710647	1149851	1860498	3010349	4870847