Fractions and Polygons

Mathematical Explorations – Math 110

Block 2, Fall 2007

1. Type into your calculator and write out the entire decimal expansions of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$, and $\frac{1}{16}$. Which ones terminate (only have finitely many decimals)? Which ones are repeating? How many digits before each repeating fraction starts repeating itself? Can you guess a pattern? For fun, try to find the pattern for $\frac{1}{89}$.

2. Try computing $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$. What pattern do you notice? Try the same with all the fractions with 13 in the denominator. What pattern arises here?

3. Find the fractions for the following repeating decimals:

$$.27272727272727272727\ldots =$$

$$.72972972972972972972\ldots =$$

$$.00990099009900990099\ldots =$$

$$.02439024390243902439\ldots =$$

4. A continued fraction is a fraction which continues infinitely in the denominator. On your calculator, approximate and guess what the following continued fractions converge to:

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}} \qquad 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ddots}}} \qquad 3+\cfrac{1}{6+\cfrac{9}{6+\cfrac{25}{6+\ddots}}}$$

5. Take some graph paper and try to draw an equilateral triangle by only points on the grid. Use Pythagorean's Theorem to find the lengths of each side. Give the coordinates and lengths of each side of either an equilateral triangle or the closest triangle to being equilateral that you could find.

6. On graph paper, draw several polygons of various sizes and compute the area (you may want to start with triangles, which have $A = \frac{1}{2}bh$.) Count the number of vertices inside the polygon. Then count the number of vertices along the edges, including the vertices of the polygon (be careful only to count points which lie *exactly* on the edge, not just close to it). Can you see any relation between the area, the number of points in the interior, and the number of points on the edges?