

Game Theory

Mathematical Explorations – Math 110

Block 2, Fall 2007

1. **Chicken.** The game of Chicken consists of two motorists driving toward each other on a collision course. Each has the option of swerving (to avoid the collision) or continuing to drive straight. If both drivers swerve, they both survive with less glory (say payoff of 3 units each). If one swerves and the other drives straight, then the chicken loses even more face while the straight driver receives all the glory (2 units to chicken, 4 to the other driver). If both drivers continue to drive straight, they both die in a blaze of glory (receive only 1 unit of payoff each). This situation has been compared to the Cuban Missile Crisis, where a tense showdown between the U.S. and the U.S.S.R. came down to who would flinch first. What is the rational strategy for each motorist in this situation? Explain your reasoning.

		Bob	
		Swerve	Straight
Alice	Swerve	(3,3)	(2,4)
	Straight	(4,2)	(1,1)

2. **Leader.** In this situation, two drivers are attempting to enter a busy stream of traffic from opposite sides of an intersection. When the cross traffic clears, each driver must decide whether to wait for the other or to go ahead. If they both wait, then both are delayed and the benefit is minimal (2 units for each). If they both go, then they crash into each other (1 unit for each). The best situation for both is if one takes the “leader” position and goes ahead, while the “follower” waits until the leader has cleared the way. The leader receives the most benefit (4 units) while the follower receives almost as much payoff (3 units). What is the rational strategy for each motorist in this situation? Explain your reasoning.

		Bob	
		Wait	Go
Alice	Wait	(2,2)	(3,4)
	Go	(4,3)	(1,1)

3. **Prisoner’s Dilemma.** Two partners in crime are placed into separate interrogation rooms. The police have enough evidence to convict each of them for 1 year. If the police get one criminal to rat the other while the other remain quiet, the rat will be set free and the quiet criminal will be sentenced to 10 years. However, if both rat each other out, they will both receive 5 year sentences. The police let both criminals know of their options and potential sentences, but do not allow them to communicate with each other. What is the rational strategy for each criminal in this situation? Explain your reasoning.

		Bob	
		Rat	Quiet
Alice	Rat	(5,5)	(0,10)
	Quiet	(10,0)	(1,1)

4. **Nash Equilibrium.** John Nash (from A Beautiful Mind) is most famous for proving the existence of *Nash Equilibrium* in almost any game. For any n -person noncooperative game (zero-sum or nonzero-sum), where each player only has finitely many choices, there exists at least one equilibrium set of strategies. That is, there is a set of strategies for each player where no player would gain by changing unilaterally (or his or her own). Find all the Nash equilibria for the following non-zero sum game.

		Bob		
		A	B	C
Alice	A	(0,0)	(25,40)	(5,10)
	B	(40,25)	(0,0)	(5,15)
	C	(10,5)	(15,5)	(10,10)