

Pascal's Triangle

Mathematical Explorations – Math 110

Block 2, Fall 2007

1. What happens when you add the numbers across the rows? What if you alternate subtracting, then adding, then subtracting, etc.? Can you give a good explanation for why these two patterns occur? (Hint: Figure out what numbers to plug into the Binomial Theorem which gives you the sum of binomial coefficients.) What do you get when you add the squares of the numbers across the rows?
2. Rewrite Pascal's triangle so that all the leftmost 1's are in one column, then continuing with the second column being the second entry in each row, etc. (see below). What numbers do you get if you add along the "diagonals" (at 45° angles up and to the right starting at each 1 along the left side)?

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
```

3. Find a method of computing the n^{th} row of Pascal's Triangle without finding all the rows above it first. See if there's a nice pattern of the ratio of consecutive numbers going from left to right. (Hint: Try the 10^{th} row first, and see if the pattern works for the other rows.)
4. What do you notice about numbers in the rows which start with prime numbers?
5. Color the even numbers in red and the odd numbers in green. What pattern do you recognize? Try coloring the numbers which are divisible by 3 on a fresh copy of Pascal's triangle. Do the same with the numbers which divisible by 5.

Check out the following websites for experimenting with other "colorings":

<http://www.shodor.org/interactivate/activities/ColoringMultiples>

<http://www.cut-the-knot.org/Curriculum/Combinatorics/LeibnitzTriangle.shtml>

CHALLENGE PROBLEM

Expand $(a + b + c)^2$, $(a + b + c)^3$, and $(a + b + c)^4$. What do you notice about the coefficients? What do you think $(a + b + c)^n$ looks like in general? For a real challenge, try to figure out an explicit formula the coefficients of $(a + b + c)^n$, $(a + b + c + d)^n$, etc. (This could be done as a final project.)