

## CSE 101 QUIZ #1 Key

For the first 2 questions, you should answer Yes/No/Can't tell and explain your reasoning.

**Q1**> Algorithm A runs in worst-case time  $f(n)$  and B runs in worst-case time  $g(n)$ .

If  $g(n) = O(f(n)(\log n))$ ,

- a) Is B faster than A for all sufficiently large values of  $n$ ? (for all  $n \geq n_0$ )
- b) Is A faster than B for all sufficiently large values of  $n$ ? (for all  $n \geq n_0$ )

**Sol'n:**  $g(n) = O(f(n)(\log n))$  so there is positive constant  $c$ , such that  $g(n) \leq c \cdot f(n) \cdot \log n$ . However it's not possible to conclude that either  $g(n) \leq f(n)$  or  $f(n) \leq g(n)$ .

Example for  $f(n) \leq g(n)$ :  $f(n) = n$ ,  $g(n) = n \cdot \log n$

Example for  $g(n) \leq f(n)$ :  $f(n) = n$ ,  $g(n) = \log n$

Note that in the two examples above  $g(n) = O(f(n)(\log n))$  is satisfied.

**Q2**> Is  $\log 2^n = \theta(\log 3^n)$ ?

**Sol'n:**  $\log 2^n = n \cdot \log 2$  and  $\log 3^n = n \cdot \log 3$ .

We should find positive constants  $c_1$  and  $c_2$  such that,

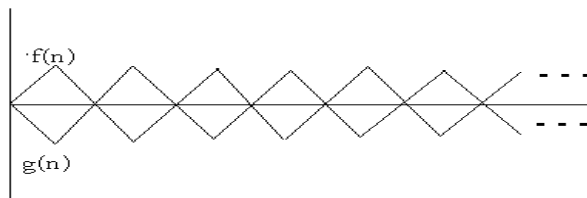
$c_1 \cdot n \cdot \log 2 \leq n \cdot \log 2 \leq c_2 \cdot n \cdot \log 3$ , for all  $n \geq n_0$  so  $c_2 \geq (\log 2 / \log 3)$ ,  $c_2$  can be set to 1 for example. Similarly,  $c_1 \leq (\log 2 / \log 3)$ ,  $c_1$  can be set to  $1/2$  for example.

Therefore,  $\log 2^n = \theta(\log 3^n)$

**Q3**> Prove or disprove the following:

For all functions  $f(n)$  and  $g(n)$ , either  $f(n) = O(g(n))$  or  $g(n) = O(f(n))$ .

**Sol'n:** Suppose we have the following two functions:



In some intervals  $f(n)$  is an upper bound for  $g(n)$ , in other intervals  $g(n)$  is an upper bound for  $f(n)$ . Therefore, we can not conclude that one function is an upper bound for the other. So the claim is not true.