

CSE 101 QUIZ #1 Key

For the first 2 questions, you should answer Yes/No/Can't tell and explain your reasoning.

Q1> Algorithm A runs in worst-case time $f(n)$ and B runs in worst-case time $g(n)$.
If $g(n) = O(f(n)(\log n))$,

- a) Is B faster than A for all sufficiently large values of n ? (for all $n \geq n_0$)
- b) Is A faster than B for all sufficiently large values of n ? (for all $n \geq n_0$)

Sol'n: $g(n) = O(f(n)(\log n))$ so there is positive constant c , such that $g(n) \leq c \cdot f(n) \cdot \log n$. However it's not possible to conclude that either $g(n) \leq f(n)$ or $f(n) \leq g(n)$.

Example for $f(n) \leq g(n)$: $f(n)=n$, $g(n)=n \cdot \log n$

Example for $g(n) \leq f(n)$: $f(n)=n$, $g(n)=\log n$

Note that in the two examples above $g(n) = O(f(n)(\log n))$ is satisfied.

Q2> Is $\log 2^n = \theta(\log 3^n)$?

Sol'n: $\log 2^n = n \cdot \log 2$ and $\log 3^n = n \cdot \log 3$.

We should find positive constants c_1 and c_2 such that,

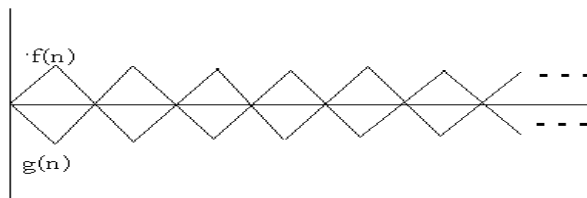
$c_1 \cdot n \cdot \log 2 \leq n \cdot \log 2 \leq c_2 \cdot n \cdot \log 3$, for all $n \geq n_0$ so $c_2 \geq (\log 2 / \log 3)$, c_2 can be set to 1 for example. Similarly, $c_1 \leq (\log 2 / \log 3)$, c_1 can be set to $1/2$ for example.

Therefore, $\log 2^n = \theta(\log 3^n)$

Q3> Prove or disprove the following:

For all functions $f(n)$ and $g(n)$, either $f(n) = O(g(n))$ or $g(n) = O(f(n))$.

Sol'n: Suppose we have the following two functions:



In some intervals $f(n)$ is an upper bound for $g(n)$, in other intervals $g(n)$ is an upper bound for $f(n)$. Therefore, we can not conclude that one function is an upper bound for the other. So the claim is not true.