Computing some constant terms by hand

Let us start with

$$\Omega_4 \;=\; \frac{1}{(1-ax)1-ay)(1-z/a)(1-w/a)} \tag{1}$$

and let us compute

$$E(x,y,z,w) \;=\; \frac{a^2}{(1-ax)1-ay)(a-z)(a-w)}\Big|_{a^0} \tag{2}$$

and

$$G(x,y,z,w) \;=\; \frac{a^2}{(1-ax)1-ay)(a-z)(a-w)} \tag{3}$$

We start by writing

$$\Omega_4 \;=\; \frac{a^2}{(1-ax)(1-ay)(a-z)(a-w)}$$

then determine A,B,C,D giviing

$$\Omega_4 \;=\; \frac{A}{1-ax} + \frac{B}{1-ay} + \frac{C}{a-z} + \frac{D}{a-w} \tag{4}$$

Now we have

$$\begin{aligned} A=(1-ax)\Omega_4\Big|_{a=1/x} &= \frac{a^2}{(1-ay)(a-z)(a-w)}\Big|_{a=1/x} \\ &= \frac{a^2}{(1-ay)(a-z)(a-w)}\Big|_{a=1/x} \\ &= \frac{1/x^2}{(1-y/x)(1/x-z)(1/x-w)} \\ &= \frac{x}{(x-y)(1-zx)(1-xw)} \end{aligned} \tag{5}$$

similarly we get

$$\begin{aligned} B=(1-ay)\Omega_4\Big|_{a=1/y} &= \frac{a^2}{(1-ax)(a-z)(a-w)}\Big|_{a=1/y} \\ &= \frac{1/y^2}{(1-x/y)(1/y-z)(1/y-w)} \\ &= \frac{y}{(y-x)(1-zy)(1-yw)} \end{aligned} \tag{6}$$

Now from (4) we get that

$$a)\quad \Omega_4\Big|_{a^0} \;=\; A+B\,, \qquad\qquad b)\quad \Omega_4\Big|_{a^{\geq}} \;=\; \frac{A}{1-x}+\frac{B}{1-y} \tag{7}$$

In fact we have

$$\begin{aligned} \frac{1}{(1-ax)}\Big|_{a^0} &= (1+ax+ax^2+\cdots)\Big|_{a^0} = 1 \\ \frac{1}{(1-ay)}\Big|_{a^0} &= (1+ay+ay^2+\cdots)\Big|_{a^0} = 1 \\ \frac{1}{(a-z)}\Big|_{a^0} = \frac{1/a}{(1-z/a)}\Big|_{a^0} &= (1/a+z/a^2+z^2/a^3+\cdots)\Big|_{a^0} = 0 \\ \frac{1}{(a-w)}\Big|_{a^0} = \frac{1/a}{(1-w/a)}\Big|_{a^0} &= (1/a+w/a^2+w^2/a^3+\cdots)\Big|_{a^0} = 0 \end{aligned}$$

This proves (7) a). To prove (7) b) we simply note that by definition "$\big|_{a\geq}$" selects the summ of the terms in a formal series where a appears to a positive power and then set $a=1$. Thus

$$\frac{1}{(1-ax)}\Big|_{a\geq} \;=\; (1+ax+ax^2+\cdots)\Big|_{a^0} \;=\; \frac{1}{1-x}$$

$$\frac{1}{(1-ay)}\Big|_{a^0} \;=\; (1+ay+ay^2+\cdots)\Big|_{a^0} \;=\; \frac{1}{1-y}$$

$$\frac{1}{(a-z)}\Big|_{a^0} \;=\; \frac{1/a}{(1-z/a)}\Big|_{a^0} \;=\; (1/a+z/a^2+z^2/a^3+\cdots)\Big|_{a^0} \;=\; 0$$

$$\frac{1}{(a-w)}\Big|_{a^0} \;=\; \frac{1/a}{(1-w/a)}\Big|_{a^0} \;=\; (1/a+w/a^2+w^2/a^3+\cdots)\Big|_{a^0} \;=\; 0$$

Using (5) and (6) in (7) we get

$$\begin{aligned}
E(x,y,z,w) &= \frac{x}{(x-y)(1-zx)(1-xw)} + \frac{y}{(y-x)(1-yz)(1-yw)} \\
&= \frac{x(1-zy)(1-yw) \,-\, y(1-zx)(1-xw)}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{x(1-zy-yw+y^2zw) \,-\, y(1-zx-xw+x^2zw)}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{x-xyz-xyw+xy^2zw \,-\, (y-zxy-xyw+x^2yzw)}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{x-xyz-xyw+xy^2zw \,-\, y+zxy+xyw-x^2yzw}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{x-y+xy^2zw-x^2yzw}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{x-y+(y-x)xyzw}{(x-y)(1-zx)(1-xw)(1-yz)(1-yw)} \\
&= \frac{1-xyzw}{(1-zx)(1-xw)(1-yz)(1-yw)}
\end{aligned}$$

$$\begin{aligned}
G(x,y,z,w) &= \frac{x}{(1-x)(x-y)(1-zx)(1-xw)} + \frac{y}{(1-y)(y-x)(1-zy)(1-yw)} \\
&= \frac{x(1-y)(1-zy)(1-yw) \,-\, y(1-x)(1-zx)(1-xw)}{(1-x)(1-y)(x-y)(1-zx)(1-xw)(1-zy)(1-yw)} \\
&= \frac{1-wxy-xyz-wxyz+wx^2yz+wxy^2z}{(1-x)(1-y)(1-zx)(1-xw)(1-zy)(1-yw)}
\end{aligned}$$