Math 10C, Spring 2001 May 30, 2001
Prof. Tesler

Homework #10, Due Thursday, June 7

10.5# 41, 43
and the problems below.

The problems below are based on the lectures given Friday June 1 and Monday June 4.

Problem H-14. Let $\vec{a} = \begin{bmatrix}5\\0\\4\end{bmatrix}$, $\vec{b} = 7\hat{\imath} - 2\hat{\jmath} + 9\hat{k}$, $\vec{c} = 5\hat{\jmath} - 3\hat{\imath}$.
(Be sure to rewrite $\vec{c}$ in the standard way when using it.)

(i) Write $\vec{a}$ in $\hat{\imath}\hat{\jmath}\hat{k}$ notation, and $\vec{b}$ and $\vec{c}$ in 3D column vector notation.

(ii) Compute all of these using $\hat{\imath}\hat{\jmath}\hat{k}$ notation.
$\vec{b}+\vec{c}$, $\vec{b}-\vec{c}$, $\vec{b}\cdot\vec{c}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{b}$, $\hat{\jmath}\times(\vec{b}\times\vec{c})$, $(\hat{\jmath}\times\vec{b})\times\vec{c}$, $\vec{b}\times\vec{b}$, $10\vec{a}-3\vec{b}$.

Problem H-15. We look at the triangle with vertices at coordinates $P = (2,2,3)$, $Q = (1,5,3)$, $R = (1,2,7)$ one last time.

(i) Compute the formula of the plane containing the triangle.

To do this, you need to know a point in the plane (any of P, Q, or R will do), and a normal vector to the plane. Since the cross product of two vectors is perpendicular to both of them, and since $\overrightarrow{PQ}$, $\overrightarrow{PR}$ are two vectors in the plane, a normal vector is $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$. (Other combinations of vectors between the three points P, Q, R can also be used, and will either give the same value for $\vec{n}$ or the negative of it.)

(ii) Verify that $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$ is perpendicular to $\overrightarrow{PQ}$ and also perpendicular to $\overrightarrow{PR}$ by using the dot product.

(iii) Compute the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ by using the fact that $|\vec{v}\times\vec{w}| = |\vec{v}|\,|\vec{w}|\sin\theta$, where θ is the angle between the vectors. Your answer should be in radians, to three decimal places.

(iv) Compute the area of the triangle with vertices P, Q, R as follows: the triangle is half of the quadrilateral spanned by the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. The area of that quadrilateral is $|\overrightarrow{PQ}\times\overrightarrow{PR}|$. See the picture below: the triangle we want is in white, and the grey one is the same but rotated 180°.

(v) Compute the distance from R to the line through P and Q by this method:

The area equals half of the base $|\overrightarrow{PQ}|$ times the height, h, of the triangle. It ALSO equals the formula computed in the previous question. Stick the two together to solve for the height of the triangle (which is the same as the requested distance). See the picture below.