Math 10C, Spring 2001 May 9, 2001
Prof. Tesler

Homework #7, Due Thursday, May 17

Since this homework will not be returned until after the second midterm, you should photocopy it before turning it in if you want to use it for studying.

10.5# 1–5 odd, 15–21 odd, 29, 31

(note: the book's answers say "grad f" instead of "∇f")

and the problems below.

Problem H-10. Compute the directional derivative of $f(x,y) = x^2y^3$ at $(2,4)$ in the direction towards $(6,7)$. *Be careful: this differs from the problems in the book because* $(6,7)$ *is a point, not a vector.*

Problem H-11. A bug flies on the trajectory $\vec{r}(t) = \begin{bmatrix} t^3/3 \\ t^2 \\ 2t \end{bmatrix}$ for $1 \le t \le 5$.

(i) What is the distance between the starting and ending points of the bug's journey (ignoring the actual path the bug took in-between)?

(ii) What is the actual distance the bug flew along the path? *(Integrate the speed function; note that when you get a square root of a polynomial, the polynomial is actually a perfect square, so it's easy to do the square root.)*

(iii) Which answer is larger, and why?

Problem H-12. This is another way to find the distance between a point on a line (see Homework 3). Let $P = (2,2,3)$, $Q = (1,5,3)$, $R = (1,2,7)$. Find the point on the line through P and Q that is closest to the point R as follows:

(i) Write the ordinary parametric equations of the line through P and Q: $x = a\,t + x_0$, and similarly for y and z.

(ii) The distance from R to a point (x,y,z) on the line is $\sqrt{(x-1)^2 + (y-2)^2 + (z-7)^2} =$ a function of t, by plugging in the formulas from part (i). Write this function down, and call it $f(t)$.

(iii) Find the absolute minimum of $f(t)$ by using derivatives. (Review Chapter 4.3 if necessary.) This is the minimum distance, and should agree with the answer to Homework 3, Problem H-3(vi).

(iv) Differentiating a square root is messy because of the denominator that results. The function $g(t) = f(t)^2$ should have its minimum at the same t that $f(t)$ does, because $f(t)$ and $g(t)$ are both positive functions, and $g(t)$ is the square of $f(t)$ so they both increase or both decrease together. Compute $g(t)$ and find its absolute minimum. How does this relate to the answer to the previous part?