Math 10C **Prof. Tesler** **Midterm #1 Answer Key** **April 25, 2001**

Name *Answer Key* **Student ID No.** _____

Lecture C (MWF 3:35-4:25)

	T.A.: Nick Loehr WLH 2208	**T.A.: Rob Ellis** WLH 2206
Circle the section in which you are enrolled on Thursdays:	C01 8:00-8:50 a.m.	C03 6:50-7:40 p.m.
	C02 9:05-9:55 a.m.	C04 7:55-8:45 p.m.

Do not open this booklet until instructed to do so.

Work alone.

You must show your work; if you type the formula $\sqrt{3^2+4^2}$ into your calculator and get the answer 5, you should write down $\sqrt{3^2+4^2}=5$. If relevant work is not shown, you may not get credit.

Use the space provided. If necessary, write "see other side" and continue working on the back of the same sheet.

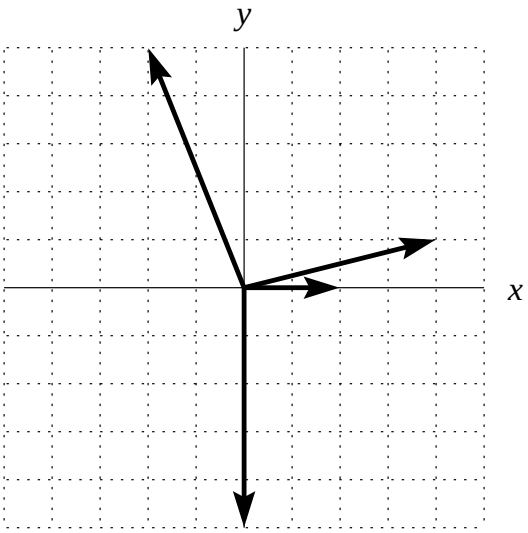
Approved calculators are permitted. No books, notes, or other resources are allowed.

Circle your final answers when relevant.

Time for midterm: 50 minutes.

1	**24**	/24
2	**24**	/24
3	**24**	/24
4	**16**	/16
5	**12**	/12
Total	**100**	/100

1. For each vector given below:
   - **Draw it** on this piece of graph paper, starting at the origin. (All four vectors will be on this same grid.)
   - In the space provided, compute its **length** and the **angle** it forms with the positive x axis (measured counterclockwise). The angle should be given in degrees, rounded to two decimal places (such as 171.39°).



The problems are in scrambled order in different versions of the test.

(a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Length $\sqrt{4^2 + 1^2} = \boxed{\sqrt{17}} \approx \boxed{4.12}$ Angle $\arctan \frac{1}{4} \approx \boxed{14.04°} \approx .245$ radians

(b) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Length $\sqrt{2^2 + 0^2} = \sqrt{4} = \boxed{2}$ Angle $\boxed{0°} = 0$ radians

(c) $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

Length $\sqrt{0^2 + (-5)^2} = \sqrt{25} = \boxed{5}$

Angle $\boxed{270°} = 3\pi/2$ radians or $\boxed{-90°} = -\pi/2$ radians

(d) $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$

Length $\sqrt{(-2)^2 + 5^2} = \boxed{\sqrt{29}} \approx 5.39$

Angle: In the quadrant it's in, angles are between 90° and 180°. $\arctan(5/(-2)) = -68.20° = -1.19$ radians as evaluated on a calculator; this is 180° away from the direction the vector is pointing, so add 180° to get $\boxed{111.80°}$ (1.95 radians).

2. Let (version X) $P = (-1, 2, 3)$, $Q = (3, -2, 5)$; (version Y) $P = (2, 3, -1)$, $Q = (-2, 5, 3)$; (version Z) $P = (3, -1, 2)$, $Q = (5, 3, -2)$.

(a) Compute the distance from P to Q.

(version X) $\overrightarrow{PQ} = \begin{bmatrix} 3-(-1) \\ (-2)-2 \\ 5-3 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

(version Y) $\overrightarrow{PQ} = \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix}$ (version Z) $\overrightarrow{PQ} = \begin{bmatrix} 2 \\ 4 \\ -4 \end{bmatrix}$

$|\overrightarrow{PQ}| = \sqrt{4^2 + (-4)^2 + 2^2} = \sqrt{16 + 16 + 4} = \sqrt{36} = \boxed{6}$

(b) Compute the unit vector in the same direction as $\overrightarrow{PQ}$.

(version X) $\vec{u} = \overrightarrow{PQ}/|\overrightarrow{PQ}| = \begin{bmatrix} 4/6 \\ -4/6 \\ 2/6 \end{bmatrix} = \boxed{\begin{bmatrix} 2/3 \\ -2/3 \\ 1/3 \end{bmatrix}}$

(version Y) $\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$ (version Z) $\begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}$

(c) On the line through P and Q, compute the coordinates of the point 2 units from P in the direction towards Q.

(version X) Add $2\vec{u} = \begin{bmatrix} 4/3 \\ -4/3 \\ 2/3 \end{bmatrix}$ to the coordinates of P:

$(-1 + 4/3, 2 - 4/3, 3 + 2/3) = \boxed{(1/3, 2/3, 11/3)} \approx (.33, .67, 3.67)$.

(version Y) $\boxed{(2/3, 11/3, 1/3)}$ (version Z) $\boxed{(11/3, 1/3, 2/3)}$

3. Let $\vec{p} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and (version X) $\vec{q} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$; (version Y) $\vec{q} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$; (version Z) $\vec{q} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.
Use these values for all questions on this page.

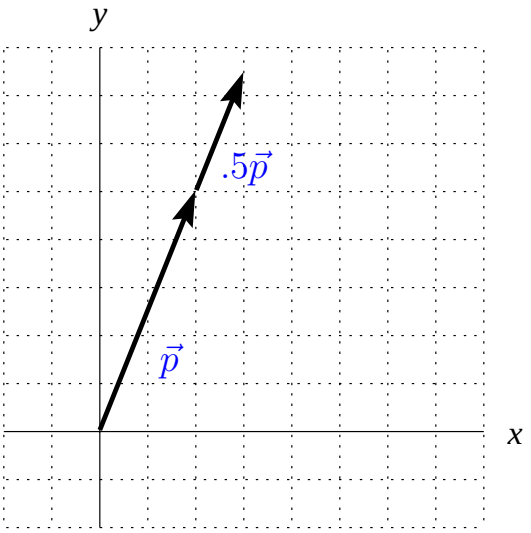
(a) Compute $10\vec{p} - 4\vec{q}$.

(version X) $10\vec{p} - 4\vec{q} = \begin{bmatrix} 10(2) \\ 10(5) \end{bmatrix} - \begin{bmatrix} 4(3) \\ 4(-1) \end{bmatrix} = \begin{bmatrix} 20 \\ 50 \end{bmatrix} - \begin{bmatrix} 12 \\ -4 \end{bmatrix} = \begin{bmatrix} 20-12 \\ 50-(-4) \end{bmatrix} = \boxed{\begin{bmatrix} 8 \\ 54 \end{bmatrix}}$

(version Y) $10\vec{p} - 4\vec{q} = \begin{bmatrix} 10(2) \\ 10(5) \end{bmatrix} - \begin{bmatrix} 4(4) \\ 4(-1) \end{bmatrix} = \begin{bmatrix} 20 \\ 50 \end{bmatrix} - \begin{bmatrix} 16 \\ -4 \end{bmatrix} = \begin{bmatrix} 20-16 \\ 50-(-4) \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ 54 \end{bmatrix}}$

(version Z) $10\vec{p} - 4\vec{q} = \begin{bmatrix} 10(2) \\ 10(5) \end{bmatrix} - \begin{bmatrix} 4(3) \\ 4(-2) \end{bmatrix} = \begin{bmatrix} 20 \\ 50 \end{bmatrix} - \begin{bmatrix} 12 \\ -8 \end{bmatrix} = \begin{bmatrix} 20-12 \\ 50-(-8) \end{bmatrix} = \boxed{\begin{bmatrix} 8 \\ 58 \end{bmatrix}}$

(b) Draw a picture illustrating how to compute $1.5\vec{p}$ from $\vec{p}$.



Numerically: $1.5\vec{p} = \begin{bmatrix} 1.5 \cdot 2 \\ 1.5 \cdot 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 7.5 \end{bmatrix}$

(c) The vector $\begin{bmatrix} 10 \\ -4 \end{bmatrix}$ is perpendicular to either $\vec{p}$ or to $\vec{q}$. Say which one, and prove it.

It's perpendicular to $\boxed{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}$ because $\begin{bmatrix} 10 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} = (10)(2) + (-4)(5) = 20 - 20 = 0.$

4. A triangle has vertices at points $A = (2, 1, 3)$, $B = (1, 0, -3)$, $C = (1, -1, 4)$.
   (a) Find the angle at B. Express it in radians to two decimal places.

$$\overrightarrow{BA} = \begin{bmatrix} 2-1 \\ 1-0 \\ 3-(-3) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix} \qquad \overrightarrow{BC} = \begin{bmatrix} 1-1 \\ -1-0 \\ 4-(-3) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 7 \end{bmatrix}$$

$$\begin{aligned} \cos\theta &= \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}|\,|\overrightarrow{BC}|} \\ &= \frac{(1)(0)+(1)(-1)+(6)(7)}{\sqrt{1^2+1^2+6^2}\sqrt{0^2+(-1)^2+7^2}} \\ &= \frac{0-1+42}{\sqrt{1+1+36}\sqrt{0+1+49}} = \frac{41}{\sqrt{38}\sqrt{50}} = \frac{41}{\sqrt{1900}} \end{aligned}$$

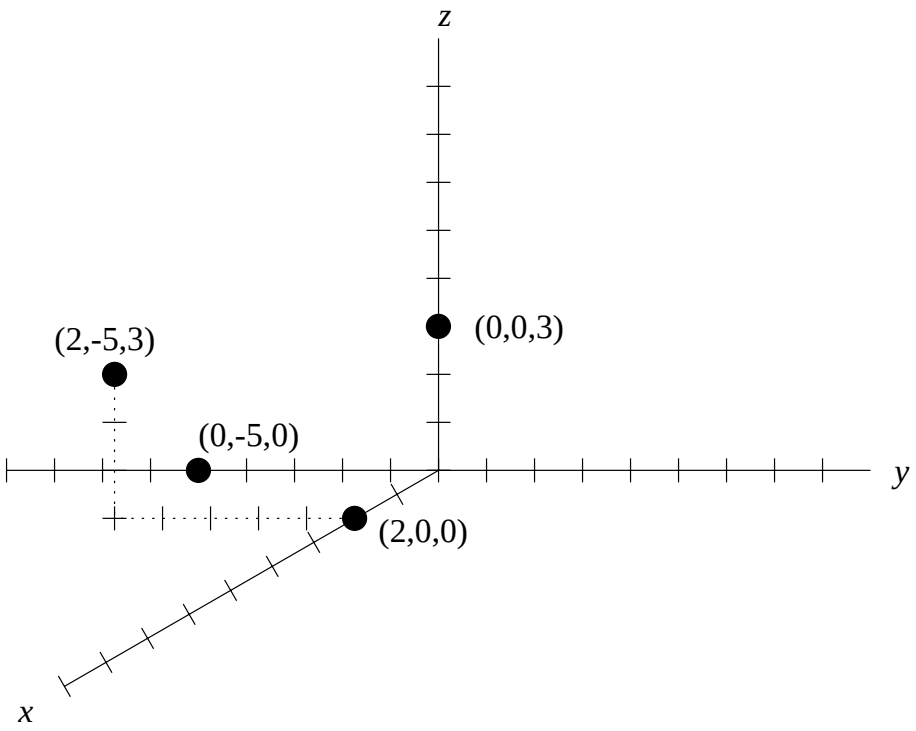
$$\theta = \arccos\left(\frac{41}{\sqrt{1900}}\right) \approx \boxed{.35 \text{ radians}} \approx 19.85^\circ$$

   (b) Give a vector parametric equation ($\vec{r}$ = some function of t) of the line through A and B. Setting $t = 0$ should correspond to the point A, and setting $t = 1$ should correspond to B.

The vector parametric equation of a line is $\vec{r} = \vec{r}_0 + t\vec{v}$.

Set $\vec{r}_0 = \overrightarrow{0A} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \overrightarrow{AB} = \begin{bmatrix} -1 \\ -1 \\ -6 \end{bmatrix}$: $\quad \boxed{\vec{r} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ -6 \end{bmatrix}}$

5. Draw 3D axes, properly labeled and scaled. Plot the points $(2, 0, 0)$, $(0, -5, 0)$, $(0, 0, 3)$, and $(2, -5, 3)$ on this set of axes.



Did you remember to

- **Put your name, student ID number, and section number on the front?**
- **Circle your final answers?**
- **Check your work?**

Exam booklets will be collected promptly when time is called.