Math 20C
Midterm Exam 2
March 1, 2013

Version A

Instructions

1. No calculators or other electronic devices are allowed during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your Name, PID, and Section on the front of your Blue Book.
4. Write the Version of your exam at the top of the page on the front of your Blue Book.
5. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new side of a page.
6. Read each question carefully, and answer each question completely.
7. Show all of your work; no credit will be given for unsupported answers.

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Let \( f(x, y) = x^3 + y^3 + 3x^2 - 12y + 6 \). Find all critical points of \( f \). Classify each critical point as a local maximum, local minimum, or saddle point. Compute the value of \( f \) at each critical point.

2. Let \( f(x, y) = x^2 + y^3 + xy + 21 \).
   (a) (4 points) Find an equation for the tangent plane to the surface \( z = f(x, y) \) at the point where \( x = 3 \) and \( y = -1 \).
   (b) (2 points) Use the linear approximation to \( f(x, y) \) at \( (3, -1) \) to estimate the value of \( f(3.1, -1.2) \). *Hint: This makes use of your answer to (b) and is not the exact value of \( f(3.1, -1.2) \).*

3. (6 points) Find the global maximum and minimum values of \( f(x, y) = 10 + 2x - x^2 - y^2 \) on the domain \( D = \{(x, y) \mid x^2 + y^2 \leq 4\} \).

4. (6 points) A light is turned on near the lair of a wolf spider. The brightness of the light is \( B(x, y) = \frac{18}{1 + x^2 + y^2} \) lumens, with \( x \) and \( y \) in centimeters. To get away from the light, the spider crawls along a path \( \mathbf{r}(t) \) so that 2 seconds later, its position is \( \mathbf{r}(2) = \langle 2, 2 \rangle \) centimeters and its velocity is \( \mathbf{r}'(2) = \langle 7, 5 \rangle \) centimeters/second. Find the rate of change of brightness in lumens/second along the spider’s path at time \( t = 2 \) seconds.