

Math 10A
Final Examination
December 6, 2011
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Version A

Instructions

1. No calculators or other electronic devices are allowed during this exam.
 2. You may use one page of notes, but no books or other assistance during this exam.
 3. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.
 4. Write the *Version* of your exam at the top of the page on the front of your Blue Book.
 5. Write your solutions clearly in your Blue Book
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each question on a new side of a page.
 6. Read each question carefully, and answer each question completely.
 7. Show all of your work; no credit will be given for unsupported answers.
0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.
1. (12 points) Evaluate the following limits. If the limit does not exist, write “does not exist”. Please justify your answers.

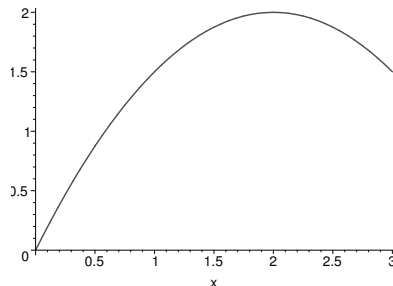
(a) $\lim_{x \rightarrow 0} \frac{x^2 + 3x}{2x^2 - 1}$

(b) $\lim_{x \rightarrow \infty} \sin(x)$

(c) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

2. (12 points) Let f be a differentiable function with $f(3) = 2$ and $f'(3) = -5$.
- (a) Let $p(x) = \ln(f(x))$. Find $p'(3)$.
 - (b) Let $q(x) = \frac{f(x)}{x}$. Find $q'(3)$.

3. (9 points) The graph of a function f is given below. For each of the following pairs of numbers, determine which is greater. No justification is required.



- (a) $f'(1)$, $f'(2)$
 (b) $\frac{f(1) - f(0)}{1 - 0}$, $\frac{f(2) - f(1)}{2 - 1}$
 (c) $f'(2)$, $\frac{f(2) - f(1)}{2 - 1}$
4. (12 points) A function f is given by:

$$f(x) = \begin{cases} b & \text{if } x < -2, \\ x^2 + 4x & \text{if } -2 \leq x \leq 0, \\ ax & \text{if } x > 0. \end{cases}$$

where a and b are fixed constants.

- (a) For which value(s) of a is f differentiable at $x = 0$?
 (b) For which value(s) of b is f differentiable at $x = -2$?
5. (a) (10 points) Find an equation for the tangent line to the graph $y = e^{\frac{x}{3}}$ at $x = 0$.
 (b) (5 points) Using calculus (and part (a)), explain why $\frac{4}{3} < e^{\frac{1}{3}}$.
6. (8 points) Given $\cos(xy) = y^2$, find $\frac{dy}{dx}$ using implicit differentiation.
7. (15 points) Find all the critical points of $f(x) = x^2 e^{-x}$ and classify each critical point as a local minimum, local maximum, or neither.
8. (16 points) Farmer John has 1200 feet of fencing and wishes to use it to fence a rectangular plot divided into two subplots as in the figure below. What are the dimensions of the plot with maximum area?

