

Math 20E

August 20, 2013

Question 1 Given a path $c : [a, b] \rightarrow \mathbb{R}^n$. c is *regular* at t_0 means

A. the derivative $c'(t_0)$ exists.

***B.** the derivative $c'(t_0)$ exists and is not zero.

C. the image curve $c([a, b])$ has a tangent vector at $c(t_0)$.

D. $c'(t_0)$ is a unit vector.

E. both **B** and **C**.

Question 2 Two paths $\mathbf{c}_1 : [0, 2\pi] \rightarrow \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \rightarrow \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = (\cos(t), \sin(t), t)$$

$$\mathbf{c}_2(t) = (\cos(2\pi - t), \sin(2\pi - t), 2\pi - t)$$

The lengths of the corresponding curves are

- ***A.** the same since the curves defined by the paths are the same.
- B.** of opposite sign since the paths traverse the curves in opposite directions.
- C.** cannot be computed because the antiderivative of $\|\mathbf{c}'_1(t)\|$ and $\|\mathbf{c}'_2(t)\|$ cannot be computed.
- D.** both **B** and **C**
- E.** none of the above

Question 3 Two paths $\mathbf{c}_1 : [0, 2\pi] \rightarrow \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \rightarrow \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = (\cos(t), \sin(t), t)$$

$$\mathbf{c}_2(t) = (\cos(2\pi - t), \sin(2\pi - t), 2\pi - t)$$

Let $\mathbf{F}(x, y, z)$ be any C^1 vector field. The value of the line integrals $\int_{\mathbf{c}_1} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{c}_2} \mathbf{F} \cdot d\mathbf{s}$ are

- A.** the same since the curves defined by the paths are the same.
- ***B.** of opposite sign since the paths traverse the curves in opposite directions.
- C.** cannot be computed because the antiderivative of $\|\mathbf{c}'_1(t)\|$ and $\|\mathbf{c}'_2(t)\|$ cannot be computed.
- D.** both **B** and **C**
- E.** none of the above