Math 20E

August 22, 2013

Question 1 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (2u, 2v, 1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface S has a tangent plane at every point.

***D.** **A**, **B** and **C**

E. none of the above

Question 2 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(u, v) = (u\cos(v), u\sin(v), 1 - u^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = u(2u\cos(v), 2u\sin(v), u)$ and

A. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

B. The parametrized surface Ψ is regular at every point of S.

***C.** The surface S has a tangent plane at every point.

D. **A**, **B** and **C**

E. none of the above

Question 3 The surface S given by $x^2+y^2+z=1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2+v^2 \leq 1$ and $\Phi(u,v) = (u, v, 1-u^2-v^2)$. S is also parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0,1] \times [0, 2\pi]$ and $\Psi(r,\theta) = (r\cos(\theta), r\sin(\theta), 1-r^2)$.

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

C. $\Psi = \Phi \circ T$, where $T : R \to D$ is the polar coordinate transformation.

D. **A**, **B** and **C**

***E.** **A** and **C**