Math 20E

August 26, 2013

Question 1 Given a C^1 surface S parameterized by $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))\,,$$

let $f(x,y,z)$ be a continuous function defined on S. Then,

A. $\displaystyle\iint_S f\,dS = \iint_D f\,(\Phi\,(u,v))\,||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv$

B. $\displaystyle\iint_S f\,dS = \iint_D f\,(\Phi\,(u,v))\,||\mathbf{T}_v \times \mathbf{T}_u||\,dv\,du$

C. The average value of f on S is $\displaystyle\frac{1}{A(S)}\iint_S f\,dS$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv$$

D. **A** and **B**: they are the same

***E.** **A**, **B** and **C**

Question 2 Given a C^1 surface with two distinct parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, then

***A.** $\iint_\Phi f\, dS = \iint_\Psi f\, dS$

B. $\iint_\Phi f\, dS < \iint_\Psi f\, dS$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$

C. $\iint_\Phi f\, dS = -\iint_\Psi f\, dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = -\left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\right)$

D. **B** and **C**

E. none of the above

Question 3 Given a C^1 surface S parameterized by $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))\,,$$

and given $\mathbf{F}(x,y,z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_u \times \mathbf{T}_v\right)\, du\, dv$

B. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = -\iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_v \times \mathbf{T}_u\right)\, dv\, du$

C. The average value of $\mathbf{F}$ on S is $\displaystyle\frac{1}{A(S)}\iint_S \mathbf{F}\cdot d\mathbf{S}$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv$$

***D.** **A** and **B**

E. **A**, **B** and **C**

Question 4 Given a C^1 surface with two distinct parameterizations $\Phi : D \to S$ and $\Psi : D \to S$, then

A. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$

B. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $\dfrac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}||} = \dfrac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||}$

C. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = -\iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $\dfrac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}||} = -\dfrac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||}$

D. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} < \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$

*__E.__ **B** and **C**