

Math 142B
Midterm Exam 1
April 20, 2012
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Instructions

1. You may use any type of calculator, but no other electronic devices during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.
4. Write your solutions clearly in your Blue Book
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each question on a new side of a page.
5. Read each question carefully, and answer each question completely.
6. Show all of your work; no credit will be given for unsupported answers.

1. Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ -1 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Show that f is not integrable on $[0, 1]$.
 - (b) Show that $|f|$ is integrable on $[0, 1]$.
2. Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \frac{1}{2}, \\ 2 & \text{if } \frac{1}{2} < x \leq 1. \end{cases}$$

- (a) Let P_n be the n^{th} regular partition of $[0, 1]$ into n partition intervals. Show that $\{P_n\}$ is Archimedean for f on $[0, 1]$.
 - (b) Determine the value of $\int_0^1 f$.
3. Let $f : (-1, 1) \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{1-x^2}$ and $g : (-1, 1) \rightarrow \mathbb{R}$ be defined by $g(x) = \sin\left(\frac{1}{1-x^2}\right)$.
- (a) Extend f to $f : [-1, 1] \rightarrow \mathbb{R}$ by defining $f(-1) = f(1) = 0$. Is f integrable on $[-1, 1]$? Explain.
 - (b) Extend g to $g : [-1, 1] \rightarrow \mathbb{R}$ by defining $g(-1) = g(1) = 0$. Is g integrable on $[-1, 1]$? Explain.
4. Let $f : [a, b] \rightarrow \mathbb{R}$ be monotonically *decreasing*.
- (a) Show that f is bounded on $[a, b]$.
 - (b) Let P_n be the n^{th} regular partition of $[a, b]$ into n partition intervals. Show that $\{P_n\}$ is Archimedean for f on $[a, b]$.