Math 142B
Midterm Exam 2
May 18, 2012
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Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
5. **Read each question carefully, and answer each question completely.**
6. **Show all of your work; no credit will be given for unsupported answers.**

1. Given $f : [a, b] \to \mathbb{R}$ continuous. Use the Second Fundamental Theorem and additivity of the integral to show that
$$\frac{d}{dx}\left[\int_x^b f\right] = -f(x) \quad \text{for all } x \text{ in } (a, b).$$
(Note: The *definition* that $\int_b^a f = -\int_a^b f$ is based on this result. Thus, your proof should not appeal to this definition.)

2. Given a polynomial p of degree at most n and x_0 any point. Show that the n^{th} Taylor polynomial for p at x_0 is p itself. You may assume that p can be written in the form $p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots + a_n(x - x_0)^n$.

3. Given the function $f : [0, 1] \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le \frac{1}{2}, \\ 2 & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$
As we've seen, f is integrable and $\int_0^1 f = \frac{3}{2}$. Show that the Mean Value Theorem for Integrals does not hold for f and explain why f does not satisfy the hypotheses for the Mean Value Theorem for Integrals.

4. Given a number r with $0 < r < 1$, let $f : [-r, r] \to \mathbb{R}$ be defined by $f(x) = (1 - x)^{-1}$.
   (a) Find a formula for the n^{th} Taylor polynomial for f at 0.
   (b) Use the Lagrange Remainder Theorem to show that the Taylor series for f at 0 converges to $f(x)$ for all $x \in [-r, r]$.