# Math 20E

August 19, 2014

**Question 1** Given a parametrized surface $\Phi : D \to \mathbb{R}^3$. $\Phi$ is *regular* at $(u_0, v_0)$ means

**A.** the vector $\mathbf{T}_u \times \mathbf{T}_v$ is normal to the surface $S = \Phi(D)$ at $(u_0, v_0)$.

***B.** the vector $\mathbf{T}_u \times \mathbf{T}_v$ is not zero at $(u_0, v_0)$.

**C.** the surface $S = \Phi(D)$ has a tangent plane at $\Phi(u_0, v_0)$.

**D.** $\mathbf{T}_u \times \mathbf{T}_v$ at $(u_0, v_0)$ is a unit vector.

**E.** both **B** and **C**, since **C** follows from **B**.

**Question 2** The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where $D$ is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (2u, 2v, 1)$ and

**A.** $\Phi$ is a one-to-one mapping of $D$ onto $S = \Phi(D)$.

**B.** The parametrized surface $\Phi$ is regular at every point of $S$.

**C.** The surface $S = \Phi(D)$ has a tangent plane at every point of $S$.

*__D.__ **A**, **B** and **C**

**E.** none of the above

**Question 3** The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where $R$ is the rectangle $[0,1] \times [0,2\pi]$ and $\Psi(u,v) = (u\cos(v), u\sin(v), 1-u^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = u(2u\cos(v), 2u\sin(v), u)$ and

**A.** $\Psi$ is a one-to-one mapping of $R$ onto $S = \Psi(R)$.

**B.** The parametrized surface $\Psi$ is regular at every point of $S$.

***C.** The surface $S = \Psi(R)$ has a tangent plane at every point of $S$.

**D.** **A**, **B** and **C**

**E.** none of the above