Math 20E

August 21, 2014

Question 1 Let S be a C^1 surface parameterized by $\Phi : D \to S$, where

$$\Phi(u, v) = (x\,(u, v)\,,\, y\,(u, v))\,,\, z\,(u, v))\,.$$

A. $\mathbf{T}_u = \frac{\partial \Phi}{\partial u}$ and $\mathbf{T}_v = \frac{\partial \Phi}{\partial v}$ are vectors tangent to S.

B. $\mathbf{T}_u \times \mathbf{T}_v$ is a vector normal to S.

C. The area of $\Phi([u, u + \Delta u] \times [v, v + \Delta v])$ on S is approximately $||\mathbf{T}_u(u, v) \times \mathbf{T}_u(u, v)||\, \Delta u\, \Delta v$.

D. **A** and **B**

*__E.__ **A**, **B** and **C**

Question 2 Given a C^1 surface S parameterized by $\Phi : D \to S$, where

$$\Phi(u,v) = (x(u,v), y(u,v)), z(u,v)),$$

let $f(x,y,z)$ be a continuous function defined on S. Then,

A. $\iint_S f\,dS = \iint_D f(\Phi(u,v))\,||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv$

B. $\iint_S f\,dS = \iint_D f(\Phi(u,v))\,||\mathbf{T}_v \times \mathbf{T}_u||\,dv\,du$

C. The average value of f on S is $\dfrac{1}{A(S)}\iint_S f\,dS$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv$$

D. **A** and **B**: they are the same

***E.** **A**, **B** and **C**

Question 3 Given a C^1 surface with two distinct parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, then

*__A.__ $\iint_\Phi f\, dS = \iint_\Psi f\, dS$

B. $\iint_\Phi f\, dS < \iint_\Psi f\, dS$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$

C. $\iint_\Phi f\, dS = -\iint_\Psi f\, dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = -\left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\right)$

D. **B** and **C**

E. none of the above