## Math 20E

## August 27, 2014

**Question 1** Given a simple domain D with  $C^1$  boundary  $\partial D$ , the area of D is given by

$$\mathbf{A.} \ A(D) = \iint_D dx \, dy$$

**B.** 
$$A(D) = -\int_{\partial D} y \, dx$$

**C.** 
$$A(D) = \frac{1}{2} \int_{\partial D} x \, dy - y \, dx$$

- D. A and C
- \*E. A, B and C

**Question 2** Consider the following domain *D*:



- **A.** By Green's theorem, the area of *D* may be computed by evaluating  $\frac{1}{2} \int_{\partial D} x \, dy y \, dx$ .
- **B.** Green's theorem cannot be applied on D since D is not a simple region.
- **C.** The area of *D* could be measured by tracing  $\partial D$  with a planimeter.
- \*D. A and C
- E. B and C

**Question 3** Given a  $C^1$  surface S parameterized by  $\Phi: D \to S$ , and given  $\mathbf{F}(x, y, z)$  a continuous vector field defined on S. Then,

**A.**  $\iint_{S} \mathbf{F} \cdot d\mathbf{S} = \iint_{S} \mathbf{F} \cdot \mathbf{n} \, dS$ , where **n** is the unit normal vector at each point of *S*.

- **B.** The unit normal vector **n** at each point of the parametrized surface  $\Phi$  is given by  $\mathbf{n} = \frac{\mathbf{T}_u \times \mathbf{T}_v}{||\mathbf{T}_u \times \mathbf{T}_v||}$
- **C.** The average value of the normal component of **F** on *S* is  $\frac{1}{A(S)} \iint_{S} \mathbf{F} \cdot d\mathbf{S}$ , where A(S) is the area of the surface *S*.
- D. A and B
- \*E. A, B and C