Math 20E

September 2, 2014

Question 1 Given a simple domain D with C^1 boundary ∂D, the area of D is given by

A. $A(D) = \iint_D dx\, dy$

B. $A(D) = -\int_{\partial D} y\, dx$

C. $A(D) = \frac{1}{2}\int_{\partial D} x\, dy - y\, dx$

D. **A** and **C**

*__E.__ **A**, **B** and **C**

Question 2 Suppose $\mathbf{F}$ is a C^1 vector field on the unit sphere S in $\mathbb{R}^3$. Then, $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. is 0

B. is most easily computed by parametrizing S using spherical coordinates.

C. is most easily computed by applying Stokes' theorem and computing $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$

D. cannot be computed using Stokes' theorem because the sphere S has no boundary curve ∂S

*__E.__ **A** and **C**: the line integral in **C** is 0 because the boundary curve ∂S is empty

Question 3 Suppose $\mathbf{F}$ is a C^1 vector field on $\mathbb{R}^3$. Let H be the unit hemisphere given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$, let D be the unit disk given by $z = 0$ with $x^2 + y^2 \leq 1$, and let $S = H \cup D$. Then,

A. $\partial H = \partial D$, including orientation when H and D are both oriented with the upward-pointing unit normal vector.

B. $\iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

C. $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} + \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

D. $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0$.

***E. A**, **B** and **D**