# Math 20E

September 3, 2014

**Question 1** Suppose $\mathbf{F}$ is a $C^1$ vector field on $\mathbb{R}^3$. Let $H$ be the unit hemisphere given by $x^2+y^2+z^2=1$ with $z \geq 0$, let $D$ be the unit disk given by $z=0$ with $x^2+y^2 \leq 1$, and let $S = H \cup D$. Then,

**A.** $\partial H = \partial D$, including orientation when $H$ and $D$ are both oriented with the upward-pointing unit normal vector.

**B.** $\iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

**C.** $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} + \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

**D.** $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0$.

*__E.__ **A**, **B** and **D**

**Question 2** Suppose $\mathbf{F}$ is a $C^2$ vector field on $\mathbb{R}^3$, and let $S$ be the unit sphere given by $x^2 + y^2 + z^2 = 1$. Then, $\iint_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S}$

**A.** by Stokes' Theorem is equal to $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

**B.** by Gauss's Theorem is equal to $\iiint_B \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{F}) dV$, where $B$ is the unit ball given by $x^2 + y^2 + z^2 \leq 1$.

**C.** is 0.

**D.** **A** and **B**

*__E.__ **A**, **B**, and **C**