Math 20E
Final Examination
December 9, 2013
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Version A

Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

0. (2 points) Carefully read and complete the instructions at the top of this exam sheet.

1. (a) (4 points) Compute the derivatives $\mathbf{D}\,\mathbf{f}(u,v,w)$ and $\mathbf{D}\,\mathbf{g}(x,y)$ for the functions

$$\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2 \quad \text{and} \quad \mathbf{g} : \mathbb{R}^2 \to \mathbb{R}^3$$

given by

$$\mathbf{f}(u,v,w) = \left(u^2 + w^2, uv + vw\right) \quad \text{and} \quad \mathbf{g}(x,y) = \left(x^2,\, xy,\, y^2\right)$$

   (b) (4 points) Use the chain rule to compute the derivative $\mathbf{D}(\mathbf{f} \circ \mathbf{g})(2,1)$.
   [Recall that $(\mathbf{f} \circ \mathbf{g})(x,y) = \mathbf{f}\left(\mathbf{g}\left(x,y\right)\right)$.]

2. (6 points) Let R be the region bounded by $x\,y = 1$, $x\,y = 3$, $x = 1$, and $x = 3$.
Use the change of variables $x = u$, $y = \dfrac{v}{u}$ to evaluate $\displaystyle\iint_R \frac{x\,y}{1 + x^2\,y^2}\,dx\,dy$.

3. (6 points) Evaluate the iterated integral $\displaystyle\int_{y=0}^{\pi/2}\int_{x=y}^{\pi/2} \frac{6\,y^2 \sin\left(x^2\right)}{x^2}\,dx\,dy$ by first changing the order of integration.

Note: Problems 4 – 8 are on the other side of this page.

4. (6 points) Consider the function $f(x, y, z) = x\,y\,z\,e^{2\,x\,y\,z}$.

   (a) Find $\nabla f(x, y, z)$, the gradient of f.

   (b) Evaluate the line integral $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s}$ along the path $\mathbf{c}$ given by $\mathbf{c}(t) = \left(t^2 \sin\left(\frac{\pi}{2}\right),\, t^5 \cos(2\pi t),\, 2t\right)$ for $0 \le t \le 1$.

5. (6 points) Find the area of the portion of the sphere of radius 2 given by $x^2+y^2+z^2 = 4$ for which $x^2 + y^2 \le 1$.

6. (6 points) Use Green's theorem to evaluate the line integral

$$\int_C \frac{1}{x}\,e^{x\,y}\,dx + \left(x + \frac{1}{y}\,e^{x\,y}\right)\,dy,$$

   where C is the circle parametrized by $3\,(\cos(t),\, \sin(t))$ for $0 \le t \le 2\pi$.

7. (6 points) Use Stokes' theorem to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where S is the surface $x^2 + y^2 + 3z^2 = 1$ with $z \le 0$, and $\mathbf{F}(x, y, z) = (2y,\ -2x,\ zx^3y^2)$.

8. (6 points) Let $\mathbf{F} = \dfrac{\mathbf{r}}{r^3}$, where $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $r = \|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$.

   Use Gauss's Law to evaluate the following surface integrals and clearly explain how you applied Gauss's Law to arrive at your answer.

   (a) $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the ellipsoid $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$.

   (b) $\displaystyle\iint_\Sigma \mathbf{F} \cdot d\mathbf{S}$, where Σ is the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 1$.