Math 20E

August 12, 2015

Question 1 In order for a transformation $T : R \to S$ to be a coordinate transformation that can be used to change variables in a double or triple integral, it should

A. be a one-to-one mapping mapping of R

B. map R onto S

C. both **A** and **B**

***D.** both **A** and **B**, except that would be OK if it failed to be one-to-one on parts of the boundary of R

E. none of the above: "one-to-one" and "onto" have nothing to do with coordinate transformations.

Question 2 Given domains $D \subset \mathbb{R}^2$ and $S \subset \mathbb{R}^2$ and a one-to-one transformation $T : D \to S$ that maps D onto S. Then T can be used to change variables as follows:

A. $\displaystyle\iint_S f(x,y)\,dx\,dy = \iint_D f\left(T\left(u,v\right)\right)\,\left|\text{det}\left[\mathbf{D}T\left(u,v\right)\right]\right|\,du\,dv.$

B. $\displaystyle\iint_D f(u,v)\,du\,dv = \iint_S f\left(T\left(x,y\right)\right)\,\left|\text{det}\left[\mathbf{D}T\left(x,y\right)\right]\right|\,dx\,dy.$

C. $\displaystyle\iint_D f(u,v)\,du\,dv = \iint_S f\left(T^{-1}\left(x,y\right)\right)\,\left|\text{det}\left[\mathbf{D}T^{-1}\left(x,y\right)\right]\right|\,dx\,dy.$

D. Both **A** and **B**

*__E.__ Both **A** and **C**

Note: The regions of integration of the integrals in each equation determine the required domain and range of the transformation.

- $T : D \to S$ in part **A**,
- $T : S \to D$ in part **B**,
- $T^{-1} : S \to D$ in part **C**.

Thus, **B** contradicts the definition of $T : D \to S$ given in the question.

Question 3 Consider the integral $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\,dx\,dy\,dz$ over the unit ball B given by $x^2+y^2+z^2 \le 1$.

A. It may be integrated by using the change of variables $\Phi(\rho,\theta,\phi) = (\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))$.

B. It may be written in the form $$\int_{\rho=0}^{1}\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi} e^{\rho^3}\rho^2\sin(\phi)\,d\phi\,d\theta\,d\rho.$$

C. It is an easy integral to evaluate after changing to spherical coordinates.

D. It cannot be evaluated analytically because $e^{(x^2+y^2+z^2)^{3/2}}$ does not have an elementary antiderivative.

***E.** **A**, **B**, and **C**