Math 20E

August 17, 2015

Question 1 Given a path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is *regular* at t_0 means

A. the derivative $\mathbf{c}'(t_0)$ exists.

B. the derivative $\mathbf{c}'(t_0)$ exists and is not zero.

C. the image curve $\mathbf{c}([a, b])$ has a tangent vector at $\mathbf{c}(t_0)$.

D. $\mathbf{c}'(t_0)$ is a unit vector.

*__E.__ both **B** and **C**.

Question 2 Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

The lengths of the corresponding curves are

*__A.__ the same since the curves defined by the paths are the same.

B. of opposite sign since the paths traverse the curves in opposite directions.

C. cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

D. both **B** and **C**

E. none of the above

Question 3 Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

Let $\mathbf{F}(x, y, z)$ be any C^1 vector field. The value of the line integrals $\int_{\mathbf{c}_1} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{c}_2} \mathbf{F} \cdot d\mathbf{s}$ are

A. the same since the curves defined by the paths are the same.

*__B.__ of opposite sign since the paths traverse the curves in opposite directions.

C. cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

D. both **B** and **C**

E. none of the above

Question 4 Given $f : \mathbb{R}^n \to \mathbb{R}$ a C^1 scalar-valued function and $\mathbf{c} : [a, b] \to \mathbb{R}^n$ a simple C^1 path,

A. $\mathrm{D}f(t) = \nabla f(\mathbf{c}(t)) \cdot \mathbf{c}'(t)$ by the chain rule.

B. $\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = \int_a^b \nabla f(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\, dt.$

C. The value of $\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s}$ is independent of the path $\mathbf{c}$.

D. $\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = f(\mathbf{c}(b)) - f(\mathbf{c}(a))$, which depends only on the value of f at the endpoints $\mathbf{c}(a)$ and $\mathbf{c}(b)$.

*__E.__ All of the above.