Math 20E

August 18, 2015

Question 1 Two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ are orthogonal (or perpendicular or normal) if and only if $\mathbf{v} \cdot \mathbf{w} = \mathbf{0}$. Thus,

A. the angle between two nonzero orthogonal vectors is $\frac{\pi}{2}$

B. the zero vector is orthogonal to every vector.

C. the zero vector is normal to every plane.

D. the zero vector is the only vector orthogonal to itself.

***E.** All of the above. (see page 24 of your textbook)

Question 2 Given a parametrized surface $\Phi : D \to \mathbb{R}^3$. Φ is *regular* at (u_0, v_0) means

A. the vector $\mathbf{T}_u \times \mathbf{T}_v$ is normal to the surface $S = \Phi(D)$ at (u_0, v_0).

B. the vector $\mathbf{T}_u \times \mathbf{T}_v$ is not zero at (u_0, v_0).

C. the surface $S = \Phi(D)$ has a tangent plane at $\Phi(u_0, v_0)$.

D. $\mathbf{T}_u \times \mathbf{T}_v$ at (u_0, v_0) is a unit vector.

***E.** both **B** and **C**.

Question 3 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (2u, 2v, 1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

***D.** **A**, **B** and **C**

E. none of the above

Question 4 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(u, v) = (u \cos(v), u \sin(v), 1 - u^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = u(2u \cos(v), 2u \sin(v), u)$ and

A. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

B. The parametrized surface Ψ is regular at every point of S.

***C.** The surface $S = \Psi(R)$ has a tangent plane at every point of S.

D. **A**, **B** and **C**

E. none of the above