

Math 20E

August 20, 2015

Question 1 Let S be a C^1 surface parametrized by $\Phi : D \rightarrow S$ (which we assume to be *regular*), where

$$\Phi(u, v) = (x(u, v), y(u, v), z(u, v)).$$

A. $\mathbf{T}_u = \frac{\partial \Phi}{\partial u}$ and $\mathbf{T}_v = \frac{\partial \Phi}{\partial v}$ are vectors tangent to S .

B. $\mathbf{T}_u \times \mathbf{T}_v$ is a vector normal to S .

C. The area of $\Phi([u, u + \Delta u] \times [v, v + \Delta v])$ on S is approximately $\|\mathbf{T}_u(u, v) \times \mathbf{T}_v(u, v)\| \Delta u \Delta v$.

D. **A** and **B**

***E.** **A**, **B** and **C**

Question 2 Given a C^1 surface S parametrized by $\Phi : D \rightarrow S$ (which we assume to be *regular*), where

$$\Phi(u, v) = (x(u, v), y(u, v), z(u, v)),$$

let $f(x, y, z)$ be a continuous function defined on S . Then,

A.
$$\iint_S f \, dS = \iint_D f(\Phi(u, v)) \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv.$$

B.
$$\iint_S f \, dS = \iint_D f(\Phi(u, v)) \|\mathbf{T}_v \times \mathbf{T}_u\| \, dv \, du.$$

C. The average value of f on S is $\frac{1}{A(S)} \iint_S f \, dS$, where

$$A(S) = \iint_D \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv.$$

D. **A** and **B**: they are the same.

***E.** All of the above.

Question 3 Given a C^1 surface with two distinct parametrizations $\Phi : D \rightarrow S$ and $\Psi : D \rightarrow S$, then

***A.** $\iint_{\Phi} f \, dS = \iint_{\Psi} f \, dS$

B. $\iint_{\Phi} f \, dS < \iint_{\Psi} f \, dS$ when $\|\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}\| < \|\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\|$

C. $\iint_{\Phi} f \, dS = - \iint_{\Psi} f \, dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = - \left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v} \right)$

D. **B** and **C**

E. none of the above