

Math 20E

August 24, 2015

Question 1 Given a C^1 surface with two distinct parametrizations $\Phi : D \rightarrow S$ and $\Psi : D \rightarrow S$, and given $f(x, y, z)$ a continuous function defined on S . Then,

***A.** $\iint_{\Phi} f \, dS = \iint_{\Psi} f \, dS$

B. $\iint_{\Phi} f \, dS < \iint_{\Psi} f \, dS$ when $\|\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}\| < \|\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\|$

C. $\iint_{\Phi} f \, dS = - \iint_{\Psi} f \, dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = - \left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v} \right)$

D. **B** and **C**

E. none of the above

Question 2 Given a C^1 surface S parameterized by $\Phi : D \rightarrow S$, where

$$\Phi(u, v) = (x(u, v), y(u, v), z(u, v)),$$

and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S . Then,

A. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\Phi(u, v)) \cdot (\mathbf{T}_u \times \mathbf{T}_v) \, du \, dv$

B. $\iint_S \mathbf{F} \cdot d\mathbf{S} = - \iint_D \mathbf{F}(\Phi(u, v)) \cdot (\mathbf{T}_v \times \mathbf{T}_u) \, dv \, du$

C. The average value of \mathbf{F} on S is $\frac{1}{A(S)} \iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$A(S) = \iint_D \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv$$

***D.** **A** and **B**

E. **A**, **B** and **C**

Question 3 Given a C^1 surface with two distinct parametrizations $\Phi : D \rightarrow S$ and $\Psi : D \rightarrow S$, and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S . Then,

A.
$$\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$$

B.
$$\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S} \text{ when } \frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{\left\| \frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} \right\|} = \frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{\left\| \frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v} \right\|}$$

C.
$$\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = - \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S} \text{ when } \frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{\left\| \frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} \right\|} = - \frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{\left\| \frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v} \right\|}$$

D.
$$\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} < \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S} \text{ when } \left\| \frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} \right\| < \left\| \frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v} \right\|$$

***E.** **B** and **C**