Math 20E

August 25, 2015

Question 1 Given a C^1 surface S parameterized by $\Phi : D \to S$, where

$$\Phi(u,v) = (x(u,v),\, y(u,v)),\, z(u,v)),$$

let $f(x,y,z)$ be a continuous function defined on S. Then,

A. $\displaystyle\iint_S f\, dS = \iint_D f\left(\Phi(u,v)\right) ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv$

B. $\displaystyle\iint_S f\, dS = \iint_D f\left(\Phi(u,v)\right) ||\mathbf{T}_v \times \mathbf{T}_u||\, dv\, du$

C. The average value of f on S is $\dfrac{1}{A(S)} \displaystyle\iint_S f\, dS$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv$$

D. **A** and **B**: they are the same

*__E.__ **A**, **B** and **C**

Question 2 Given a C^1 surface S parameterized by $\Phi : D \to S$, where

$$\Phi(u,v) = (x(u,v),\, y(u,v)),\, z(u,v)),$$

and given $\mathbf{F}(x,y,z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi(u,v)\right)\cdot\left(\mathbf{T}_u \times \mathbf{T}_v\right)\, du\, dv$

B. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = -\iint_D \mathbf{F}\left(\Phi(u,v)\right)\cdot\left(\mathbf{T}_v \times \mathbf{T}_u\right)\, dv\, du$

C. The average value of $\mathbf{F}$ on S is $\dfrac{1}{A(S)}\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv$$

*__D.__ **A** and **B**

E. **A**, **B** and **C**

Question 3 Given a simple domain D with C^1 boundary ∂D, the area of D is given by

A. $A(D) = \iint_D dx\, dy$

B. $A(D) = -\int_{\partial D} y\, dx$

C. $A(D) = \frac{1}{2}\int_{\partial D} x\, dy - y\, dx$

D. **A** and **C**

*__E.__ **A**, **B** and **C**