Math 20E
Final Examination
March 19, 2012
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Version A

Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet.

1. (6 points) Evaluate the following integral by first changing the order of integration.

$$\int_{y=0}^{4}\int_{x=\sqrt{y}}^{2} \cos(x^3)\,dx\,dy.$$

2. (6 points) Let R be the region bounded by

$$x+y=0 \qquad x+y=2 \qquad x-y=0 \qquad x-y=2$$

By first applying an appropriate change of variables, evaluate

$$\iint_R (x+y)e^{x^2-y^2}\,dx\,dy$$

3. (6 points) Given the vector field

$$\mathbf{F}(x,y,z) = \big(-2\sin(2x)e^{5yz}, 5z\cos(2x)e^{5yz}, 5y\cos(2x)e^{5yz}\big).$$

   (a) Find a scalar function $f(x,y,z)$ such that $\mathbf{F} = \nabla f$.
   (b) Evaluate the line integral $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$ from $(0,0,0)$ to $(\frac{\pi}{2},1,1)$ along the path $\mathbf{c}(t) = (\frac{\pi}{2}t, t^3\sin(\frac{\pi}{2}t), t^4\cos(2\pi t))$.

4. (6 points) Let S be the portion of the unit sphere with $z \geq \frac{1}{2}$.

   (a) Parametrize S. Be sure to clearly specify the domain of your parametrization.

   (b) Compute the area of S.

5. (6 points) Find the area enclosed by the path

$$\begin{aligned} \mathbf{c} :& [-\frac{\pi}{2}, \frac{\pi}{2}] \longrightarrow \mathbb{R}^2 \\ \mathbf{c}(t) =& (2\cos(t), 5\sin(2t)) \end{aligned}$$

6. (6 points) Given $\mathbf{F}(x, y, z) = (y - 2z, z - x, 2x - y)$. Compute the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{c}$ is the square in the xy-plane with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$, traversed in that order.

7. (6 points) Given $\mathbf{F}(x, y, z) = (x + yz, -y + xz, 3z + xy)$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the sphere $x^2 + y^2 + z^2 = 4$