Math 20F
Midterm Exam 2
February 25, 2014
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Version A

Instructions

1. **No calculators or other electronic devices are allowed during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions given during the exam.

1. (6 points) Given the 4×5 matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 3 \\ 2 & 4 & 1 & -3 & 1 \\ 1 & 2 & 4 & 2 & 1 \\ 3 & 6 & 5 & -1 & 2 \end{bmatrix}$.

   A is row equivalent to the matrix $B = \begin{bmatrix} 1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   Find a basis for each of the following subspaces associated with the matrix A:

   (a) (1 point) $\text{Col}(A)$, the column space of A.
   (b) (1 point) $\text{Row}(A)$, the row space of A.
   (c) (3 points) $\text{Nul}(A)$, the null space of A.
   (d) (1 point) $\text{Col}\left(A^T\right)$, the column space of A^T.

Exam continues with Problems 2 – 4 on the other side of this sheet.

2. (6 points) Given the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$. A can be row-reduced to the identity matrix with a sequence of three elementary row operations as follows:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Write down the elementary matrices E_1, E_2, E_3 corresponding to the elementary row operations 1, 2, 3 indicated above.

(b) Express A^{-1} as a product of elementary matrices.

(c) Compute A^{-1}.

3. (6 points) Let A be a $m \times n$ matrix and B a $n \times p$ matrix. Suppose that the $m \times p$ matrix $C = AB$ has the property that $C\mathbf{x} = \mathbf{0}$ has only the trivial solution. Find $\dim \mathrm{Col}(B)$, the dimension of the column space of B. Be sure to justify your answer.

4. (6 points) Given $\mathcal{U}$ and $\mathcal{W}$ subspaces of a vector space V. Is the union $\mathcal{U} \cup \mathcal{W}$ a subspace of V? Why or why not?