Math 20F
Final Examination
March 18, 2014
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Version A

Instructions

1. **No calculators or other electronic devices are allowed during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions given during the exam.

1. (6 points) Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 2 & 4 & -3 \end{bmatrix}$. If A is invertible, find A^{-1}. Otherwise, explain why A is not invertible.

2. (8 points) The matrices $A = \begin{bmatrix} 3 & 1 & 11 & 8 \\ 2 & 1 & 9 & 5 \\ 0 & -1 & -5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

   (a) Explain why the nonzero rows of B form a basis for $\text{Row}(A)$, the row space of A.
   (b) Find a basis for $\text{Col}(A)$, the column space of A.
   (c) Find a basis for $\text{Nul}(A)$, the null space of A.
   (d) Find a basis for $\text{Row}(A)^{\perp}$, the orthogonal complement of the row space of A. Be sure to explain how you know that it is a basis for $\text{Row}(A)^{\perp}$.

3. (6 points) Let A be a $m \times n$ matrix and B a $n \times p$ matrix. Suppose that the columns of B are linearly dependent. Show that the columns of the $m \times p$ matrix $C = AB$ are linearly dependent. Be sure to justify your answer.

Exam continues with Problems 4 – 8 on the other side of this sheet.

4. (6 points) Given an $n \times n$ matrix A for which $A^2 = 0$; that is, A^2 is the zero matrix. Show that 0 is the only eigenvalue of A.

5. (a) (6 points) Solve the initial value problem $\mathbf{y}'(t) = A\,\mathbf{y}(t);\ \ \mathbf{y}(0) = \mathbf{c},$ where

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

   (b) (2 points) Is A diagonalizable? If yes, determine the matrix P and the diagonal matrix D for which $AP = PD$.

6. (8 points) Given a 3×3 matrix A with eigenvalues -3, 2, and 5.

   (a) Explain why A is diagonalizable.

   (b) Show that the determinant of A is equal to the product of the eigenvalues of A.

   (c) Explain why A is invertible.

7. (9 points) Given the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$

   (a) Find an orthonormal basis for the column space of A.

   (b) Factor A into a product QR, where Q is an orthogonal matrix and R is an upper triangular matrix.

8. (8 points) Consider the following system of linear equations.

$$\begin{aligned} x_1 + x_2 &= 5 \\ 2x_1 + 3x_2 &= 3 \\ 2x_1 + x_2 &= -1 \end{aligned}$$

   (a) Explain why the system is inconsistent.

   (b) Find the least-squares solution to the system.