Math 20C
Final Examination
March 18, 2013
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Version A

Instructions

1. **No calculators or other electronic devices are allowed during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam at the top of the page on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Find a parametrization (vector equation) for the line of intersection of the planes $2x - y - 3z = -2$ and $-x + y + z = 1$.

2. (6 points) Consider the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 + 8$.
   (a) Find the critical points of f.
   (b) Classify each critical point of f as a local minimum, local maximum or saddle point of f.

3. (8 points) Consider the iterated integral $\displaystyle\int_{y=0}^{2}\int_{x=y}^{2} \frac{y\cos(x)}{x^2}\,dx\,dy$.
   (a) Sketch the region of integration; clearly label each part of its boundary with the appropriate equation.
   (b) Evaluate the integral. Reverse the order of integration, if necessary.

4. (6 points) Compute $\displaystyle\iint_D \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}\,dA$, where D is the circle of radius 2, centered at the origin $(0, 0)$. *Hint: use polar coordinates.*

Note: Problems 5 – 8 are on the other side of this page.

5. (6 points) Find the points on the sphere $x^2 + y^2 + z^2 = 9$ that are closest to and farthest from the point $(6, 6, -3)$. Be sure to state which of the points is closest and which is farthest from $(6, 6, -3)$.

6. (6 points) The gradient of a function $g(x, y, z)$ at the point $(-3, 4, 5)$ is $\langle -2, 1, 2 \rangle$.

   (a) Find the values of the partial derivatives g_x, g_y, and g_z at the point $(-3, 4, 5)$.

   (b) Find the maximum rate of change of g at the point $(-3, 4, 5)$ and the unit vector in the direction which the maximum rate of change occurs.

   (c) Find the rate of change of g at the point $(-3, 4, 5)$ in the direction of the point $(-1, 8, 1)$. (*Note: "in the direction of the point* $(-1, 8, 1)$*" means "in the direction of the displacement vector from* $(-3, 4, 5)$ *to* $(-1, 8, 1)$*".*)

7. (6 points) Let $f(x, y) = \ln(2x - 3y)$.

   (a) Find an equation for the tangent plane to $z = f(x, y)$ at the point $(2, 1, 0)$.

   (b) Use a linear approximation to estimate $f(1.9, 1.1)$.

8. (8 points) A particle follows a path given by $\mathbf{r}(t) = 4\cos(t)\,\mathbf{i} + 4\sin(t)\,\mathbf{j} + 2t^2\,\mathbf{k}$.

   (a) Find the position of the particle at $t = 0$ and $t = 2$.

   (b) Find the speed of the particle as a function of t.

   (c) Express the distance the particle travels along the path for $0 \leq t \leq 2$ as an integral. You need not evaluate the integral.