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THE

POLAR PLANIMETER.

A MANUAL

BY

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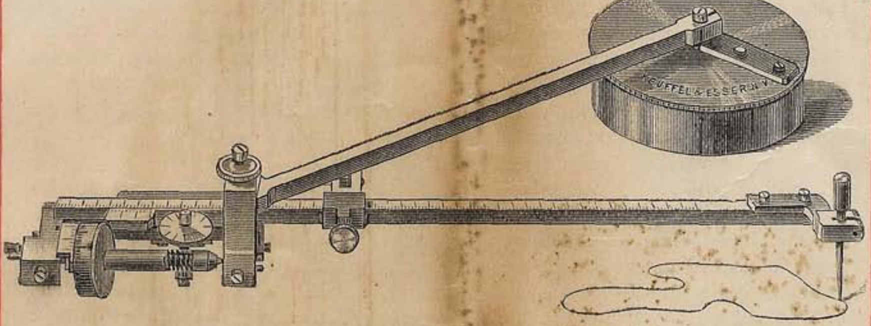


THE POLAR PLANIMETER.

BY WM. COX.

1.—Des

The Polar Planimeter is an instrument specially designed for the purpose of ascertaining the area of any plane surface (no matter how irregular its outline), such surface being represented by a figure drawn to any given scale.

It is indispensable to Engineers and others, as by it the areas of indicator diagrams, railroad profiles, plots of ground, sectional areas of vessels, areas of displacement of floating bodies, etc., may be quickly ascertained.

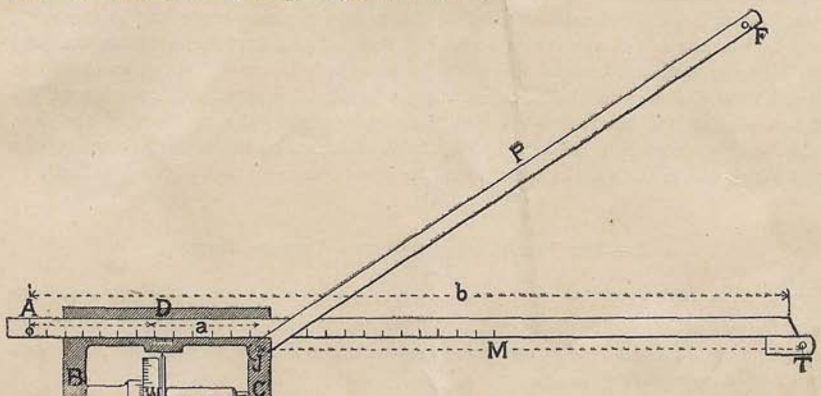


FIG. 1.

It consists of three principal parts, as shown in Fig. 1, namely: two arms J F and A T and a small wheel W, which are combined in the carriage B C D. The arm J F, called the *polar arm*, rotates about the fixed fulcrum F, and is pivoted at J to the carriage, which is thus able to revolve round the fixed point F, and at the same time to receive a circular motion around its axis J. The *movable arm* A T slides in a groove in the carriage, and can be fixed in any desired position by means of a set-screw. An adjusting screw permits its position to be regulated with great precision. At the opposite end of this arm is the tracing-point T, whose position or setting in respect to the groove (and consequently to the pivot J) depends upon the scale of the drawing whose area is to be ascertained and the value of the unit in which the result is to be expressed. The *wheel* W revolves on an axle parallel to the movable arm, in bearings in the two branches of the carriage B and C.

It will be understood that this double motion of which the arm A T is capable allows the tracer to circumscribe any figure, within of course the limits of the instrument, and that as the tracer follows the contour of the drawing, the wheel W will be made to revolve, its revolutions being registered by a small graduated disc, to which motion is communicated by means of a worm on the wheel axle. The wheel itself is divided on its circumference into ten equal parts, which are again subdivided into ten equal divisions, and, by means of a vernier, tenths of these latter can be read off, so that any number of revolutions and thousandths of a revolution may be noted. The movable arm is also graduated to half millimeters, so that the exact position in which it may be set may be indicated by means of a vernier on the carriage, giving readings to the twentieth part of a millimeter.

2.—How to Use the Instrument.

When the instrument is about to be used, the round brass weight should be placed on the table in a convenient position near the drawing whose area is required, and the fulcrum F, consisting of a ball joint, adjusted to it so that the arm J F may revolve freely. The tracer and the wheel should then rest lightly on the table, the height of the former being regulated so that there may be no strain on the pivot J.

The tracing-point is then set on the outline of the diagram or drawing, and a slight indentation made in the paper to indicate the starting-point. The movable arm having been previously adjusted to its proper position according to the value assigned to the wheel vernier unit and the scale of the drawing, and the disc and wheel being both set to zero, the tracer is then made to gently follow, as exactly as possible, and always in the direction of the hands of a watch, the outline of the figure. When the tracer has reached the starting-point, the readings of the disc, wheel and vernier are noted, and these express the area of the figure in terms of the value assigned to the wheel vernier unit (the one-thousandth part of a revolution).

3.—The Values of the Wheel Vernier Unit.

It is clear that as the pivot J acts as a fulcrum, the revolutions of the wheel will vary in proportion to its distance from J compared with the distance of the tracer, equally from J: such revolutions being greatest when J T is shortest, giving, consequently, more accurate results.

Drawings and plans are always made to a given scale, such as 1 : 10, 1 : 100, 1″= 1 ft., 1″=10 ft., etc. As any value may be given to the unit of the scale to which the drawing is made, so any value may be *assigned* to the unit of measurement of the wheel (called the wheel vernier unit or wheel unit) in relation to the actual area measured, or to a scale of 1=1, which then also expresses a ratio to the scale of the drawing. It is obvious that if the Planimeter indicated lineal measures, the ratio of its unit to the scale of the drawing would be $1 : D \times x = y$, where

- D=the scale of the drawing;
- x=the value of the wheel unit, or its ratio to a scale of 1=1, that is, to the actual size of the *object* represented by the drawing;
- y=the value *assigned* to the wheel unit, or its ratio to the scale of the *drawing* itself;

but, as the instrument registers surface measures or areas only, this ratio is expressed by the equation

$$y = D^2 \times x \quad \ldots\ldots\ldots\ldots\ldots \text{Eq. 1,}$$

from which

$$x = \frac{y}{D^2} \quad \ldots\ldots\ldots\ldots\ldots \text{Eq. 2.}$$

It is necessary to know the values of these ratios in order to ascertain exactly the position to which the movable arm should be adjusted. If a drawing were made to a scale of 1′=10 feet, and a value of 10 square feet be assigned to the wheel unit y, then each such unit marked off by the tracer traveling along the perimeter of the drawing would represent a real value of 10 square feet, and the value of the unit x, as compared with the object represented, would be 10÷100=.1 sq. foot.

If the drawing were made to the same scale of 1″=10 feet, but a value of 1 square foot be assigned to the wheel unit y, then each unit registered by the tracer would represent a real value of 1 square foot, and the value of the unit x would be 1÷100 =.01 square foot.

It will be gathered from these examples that, in order to obtain accurate, and not merely approximate, results, the value assigned to the wheel unit y should be as small as a suitable length of the arm J T will permit.

If we now suppose a drawing made to a scale of 1=1 (1 inch=1 inch), that is to say, if the *actual area* of the drawing itself be required, and each wheel unit y be taken as being equal to .01 square inch, then the value of x would also be .01 square inch, as .01÷1=.01. This is evident, as the drawing and the object represented are both on the same scale of 1=1.

It will now be clear that the wheel unit has two distinct values—an *absolute* one, or x, which is the ratio of its unit to a scale of 1=1, or of a drawing made to the actual size of the object delineated; and a *comparative* one, or y, which is the ratio of its unit to the scale of the drawing.

Herein lies the great advantage of the Polar Planimeter shown in fig. 1, as, by means of the graduated movable arm, settings of the same may be easily obtained for ascertaining the areas of drawings made to any given scale, the settings or adjustment of the movable arm being noted for reference by means of its graduations.

4.—How to obtain Settings with the Trial Disc.

These settings may be obtained in two ways, the simplest and most reliable being by means of actual tests made with a round brass trial disc furnished with each instrument, whose grooved rim encloses a known area which is indicated on the disc, and around which the tracer can be carried with exactitude.

Let, as before,

> D=the scale of the drawing;
>
> x=the *absolute* value of the wheel unit, or its ratio to a scale of 1=1;
>
> y=the *comparative* value assigned to the wheel unit, or its ratio to the scale of the drawing;
>
> W=the number of wheel units registered by the tracer making a complete circuit of the trial disc;

$$\text{then } W = \frac{\text{Area of trial disc*}}{\text{Absolute value of one unit} = x} \quad \ldots\ldots\ldots\ldots\ldots \text{ Eq. 3.}$$

When the number of wheel units is known, the exact position of the movable arm is easily found by trial, each result being noted for future reference. If the length and the graduations of the movable arm and the diameter of the wheel of every instrument were absolutely alike, a complete series of settings for various scales could be given; but as these dimensions vary slightly, a few special ones only accompany each instrument, others being obtained in the way now about to be explained. We append, however, a table which will very much facilitate the obtaining of settings for almost any scale. The figures given in the following examples, though exact for one Planimeter, may not be rigidly so for another, but they will serve to illustrate the method of procedure, and give an approximate setting by means of which the exact adjustment can be easily found.

Example: What is the setting for a drawing the scale of which is 1″=25 feet, with a value of 10 square feet assigned to each wheel unit?

$$\text{Here} \quad D=25, \quad D^2=625, \quad y=10$$

$$\text{then} \quad x = \frac{y}{D^2} = \frac{10}{625} = .016$$

$$\text{and} \quad W = \frac{\text{say } 4.075}{.016} = 255 \text{ units};$$

that is to say, the wheel must register 255 units while the tracer makes one complete circuit of the trial disc. It will then be found by trials that when the arm is set at 9, this number of units will be registered.

Example: What is the setting for a drawing to scale 1″=10″, with a wheel unit of 1 square inch?

$$D = 10, \qquad D^2 = 100, \qquad y = 1$$

$$\text{then} \quad x = \frac{1}{100} = .01$$

$$\text{and} \quad W = \frac{4.075}{.01} = 407\tfrac{1}{2} \text{ units.}$$

It will be found by trial that when the arm is set at 135.6, the wheel will register 407 units while the tracer goes once round the trial disc.

Example: What is the setting for a drawing to scale 1 : 1000, with a wheel unit of 5 square meters?

* NOTE.—The areas of the trial discs vary from 2570 to 2630 square millimeters, or 3.98½ to 4.07½ square inches.

$D = 1000$ m/m, $D^2 = 1{,}000{,}000$ sq. m/m. $y = 5$ square meters.

$$\text{then} \quad x = \frac{5{,}000{,}000}{1{,}000{,}000} = 5 \text{ square millimeters,}$$

$$\text{and} \quad W = \frac{2630}{5} = 526 \text{ units.}$$

It will be found by trial that when the arm is set at 183.3, the wheel will register 526 units while the tracer makes one complete circuit of the trial disc.

It will be seen by these examples that the higher the figure to which the arm is set, or the shorter the arm J T is, the greater will be the number of the units registered by the wheel while the tracer circumscribes the trial disc. A little practice will enable any required setting to be easily found, care being taken that the number of units comes in between 250 and 700, which are about the limits of capacity of the instrument. This is done by assigning to the wheel unit y a *suitable* value. A good working average is from 400 to 600 units for trial discs of the size which accompany these instruments, the arm being then neither too long nor too short.

We can find a suitable value for y from the equations already given, viz.:

$$y = D^2 \times x, \quad \text{and} \quad W = \frac{\text{Area trial disc}}{x}, \quad \text{from which we have}$$

$$y = \frac{D^2 \times \text{Area trial disc}}{W} \quad \ldots\ldots\ldots \text{Eq. 4.}$$

Example: Find a suitable value for the wheel unit y for a drawing to a scale of $1'' = 10$ feet.

Here $D^2 = 100$, then

$$y = \frac{100 \times 4.075}{W} = \frac{407.5}{W}$$

The value of W should be so chosen that it may divide into $D^2 \times 4.075$ without a remainder (or at least a very insignificant one), and y should be a simple number to multiply by, as 1, 2, 5, .4, .1, etc. The best number in the present case, and coming within our convenient limits of 400 and 600, is evidently 407.5; we have, therefore,

$$y = \frac{407.5}{407.5} = 1,$$

so that for a drawing to the scale of $1'' = 10$ feet, with the arm set so that the wheel shall register 407 units while the tracer goes once round the trial disc, each unit thus registered will be equivalent to one square foot.

It will frequently happen that the same setting will serve for drawings made to different scales, by assigning different values to the wheel unit y. From Equation 4,

$$y = \frac{D^2 \times \text{Area trial disc}}{W}, \quad \text{we have}$$

$$D^2 = \frac{y \times W}{\text{Area trial disc}} \quad \ldots\ldots\ldots \text{Eq. 5.}$$

Here W and the disc area are constant, and y varies. Let us take for it different values, as 4, 25, 100 (such numbers should always be taken that the square root of D^2 may be a whole number without leaving a remainder).

We now have

$$D^2 = \frac{4 \times 407}{4.07} = 400, \text{ and } D = 20$$

$$D^2 = \frac{25 \times 407}{4.07} = 2500, \text{ and } D = 50$$

$$D^2 = \frac{100 \times 407}{4.07} = 10{,}000, \text{ and } D = 100$$

so that the above setting will also serve for scales of 1″ to 20, to 50, and to 100 feet, the wheel unit values being 4, 25 and 100 square feet respectively.

Suppose, now, with a scale of 1″=25 feet, and a wheel unit of 10 square feet, the wheel, after making the circuit of a given figure, registers 2476, this number being composed of 2 on the disc, 47 on the wheel, and 6 read off by the wheel vernier, then the real area of the surface represented by the drawing will be $W \times y = 2476 \times 10 =$ 24,760 square feet.

It is not absolutely necessary that the wheel and disc be placed at zero, before beginning to follow with the tracer the outline of any figure. If they are not, the readings must be accurately noted at the commencement and at the end of a measurement, and the former be subtracted from the latter, due care being taken to add 10,000 to the second reading each time that the zero mark of the counting disc passes the fixed index mark on the carriage.

5.—How to obtain Settings by Computation.

The area of a plane figure, as obtained by passing the tracer round its perimeter, is equivalent to the area of a rectangle the side of which is equal to the length of the movable arm from the joint to the tracer =M, and its base is equal to the length of the arc rolled off by the circumference c of the wheel during W units, or $\frac{c\,W}{1000}$. We have, therefore,

$$\text{Area} = M \times \frac{c\,W}{1000} \quad \text{Eq. 6.}$$

Suppose the circumference of the wheel is 60 m/m, then

$$\text{Area} = M\ \frac{60 \times W}{1000} = .06\ M\ W$$

In order to obtain directly from W units of the above equation the area of the figure, we have merely to assign to .06 M any given value, such as 1, 10, 100 inches, feet, millimeters, etc., or any other power of 10, so that we may be enabled to obtain the length of the arm M necessary for each wheel unit rolled off to represent an area of such given number of square inches, feet, millimeters, etc.

Suppose we make .06 M=10 m/m, then we have

$$M = \frac{10}{.06} = 166\tfrac{2}{3}\ \text{m/m}$$

for the length of the arm from the joint to the tracer necessary for each wheel unit to represent an area of 10 sq. m/m, $=x$, for a drawing to a scale of 1=1. If, after the tracer has made the circuit of a figure, the wheel registers 263 units, then the area, as given above, will be .06 M W, or $.06 \times 166\frac{2}{3} \times 263 = 2630$ sq. m/m.

To find the position or the setting of the arm, we have

$$M = b - a - S \text{.................... Eq. 7.}$$

$$\text{and } S = b - a - M \text{.................... Eq. 8.}$$

where b=total length of the movable arm from zero A to the tracer T,

a=length of the movable arm from the carriage vernier zero to the joint J,

and S=length of the movable arm from zero A to the carriage vernier zero.

We have shown, Eq. 6, that the area of any figure$=M\frac{c\,W}{1000}$. If, in this equation, we substitute the value of M given in Eq. 7, we then have

$$\text{Area} = (b - a - S)\ \frac{c\,W}{1000} \text{.................. Eq. 9.}$$

and for one entire revolution of the wheel,

$$\text{Area} = (b - a - S)\ \frac{c\ 1000}{1000} = (b - a - S)\ c \text{..... Eq. 10.}$$

$$\text{and hence } c = \frac{\text{Area}}{b-a-S} \text{........................ Eq. 11.}$$

$$\text{and } S = (b - a) - \frac{\text{Area}}{c} \text{.................. Eq. 12.}$$

It must always be remembered that, as the graduations on the arm represent half millimeters, the value of S thus found must be multiplied by two to obtain the setting of the arm to the carriage vernier zero.

If we have by means of the trial disc obtained for any given instrument two settings, we can compute by means of Equation 11 the value of $b-a$ and of c, and then by means of Equation 12 we can calculate the setting for any other value of the wheel unit W, and for any other scale of drawing D.

Example: Suppose S=19.1=9.55 m/m for a scale of 1 : 1000, in which the absolute value x of the wheel unit is 10 sq. m/m, and the comparative value y of the wheel unit is 10 sq. meters:– and suppose S′=216.3=108.15 m/m for a scale of 1 : 500, in which the absolute value x of the wheel unit is 4 sq. m/m, and the comparative value y of the wheel unit is 1 sq. meter, then

For an entire revolution of the wheel, S represents an area $1000 \times 10 = 10{,}000$ sq. m/m, and S′ represents an area of $1000 \times 4 = 4{,}000$ sq. m/m. Substituting these values into Equation 11, we have

$$c = \frac{10{,}000}{(b-a)-9.55} \qquad \text{and } c = \frac{4000}{(b-a)-108.15}$$

$$\text{from which we have } \frac{10{,}000}{(b-a)-9.55} = \frac{4000}{(b-a)-108.15}$$

$$\text{and } 10{,}000\,x - 1{,}081{,}500 = 4000\,x - 38{,}200 \quad \text{and} \quad 6000\,x = 1{,}043{,}300$$

$$\text{whence } b-a \text{ or } M+S = 173.88 \text{ m/m.}$$

Inserting this value of $b-a$ in Equation 11, we get

$$c = \frac{10.000}{173.88-9.55} = \frac{10{,}000}{164.33} = 60.85 \text{ m/m.}$$

Suppose we now wish to find the setting for a scale of 1 : 1000, in which the wheel unit y is assumed as being 5 sq. meters, that is to say, with an absolute value x of 5 sq. m/m, then one entire revolution of the wheel will represent an area of 5000 sq. m/m. Introducing now the above found values for $b-a$ and c into Equation 12, we have

$$S = 173.88 - \frac{5000}{60.85} = 173.88 - 82.17 = 91.71$$

And as the graduations on the movable arm are half millimeters, we have for the setting $91.71 \times 2 = 183.4$.

Having thus obtained the values c and $b-a$, which are fixed quantities or constants for each Planimeter, and being able to obtain the value of M for any setting, seeing that by Equation 7 $M = b-a - S$, we can at once calculate the area of any plane figure by Equation 6, where Area $= M \frac{c W}{1000}$.

It will be noted that the value of M+S is a fixed quantity for each instrument, the relative value of each section of the arm M or S varying according to the setting.

Our explanations, so far, have special reference to the case of drawings whose size permits of their perimeter being circumscribed by the tracer with the fulcrum F *outside* the figure whose area is required. We shall now proceed to examine the case of larger drawings necessitating the fulcrum being placed within the area of the drawing itself.

6.—Areas of Large Surfaces necessitating the placing of the Fulcrum within the Area of the Drawing.

In obtaining such areas, the point J, Fig. 2, of intersection of the two arms will describe a *complete* circle around the fixed point F, and not merely an arc of varying

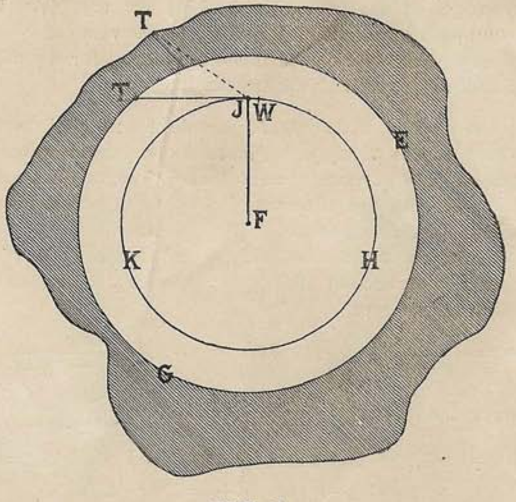


FIG. 2.

length. If the movable arm J T were fixed exactly at right angles to an imaginary line F W drawn from the fixed fulcrum to the flange of the wheel, or at a tangent to the circle H K W, described by the wheel W, it is evident that, as the tracer is drawn round, the wheel would not revole at all, its axis being all the time in the direct line of motion. A circle E G T would therefore be circumscribed by the tracer T, with a radius equal to the distance F T, of which the wheel would give no indication. Any divergence from the line of this circle would, however, cause the wheel to revolve, and that

1stly, in a *forward* direction, if the perimeter of the figure to be measured lie outside of the circle E G T, and

2ndly, in a *backward* direction, if the perimeter of the figure lie within the circumference of the circle E G T.

The wheel would therefore measure in the first case *positive* units, and in the latter case *negative* units, and in the case of an irregular figure, the difference between the two. Special care must be always taken to note if the final result be positive or negative.

It is evident, therefore, that the wheel registers only that portion of the figure which lies entirely beyond the circle E G T, as shown by the shading in Fig. 2; or the difference between the external and the internal portions, as shown by the two shaded parts in Fig. 3.

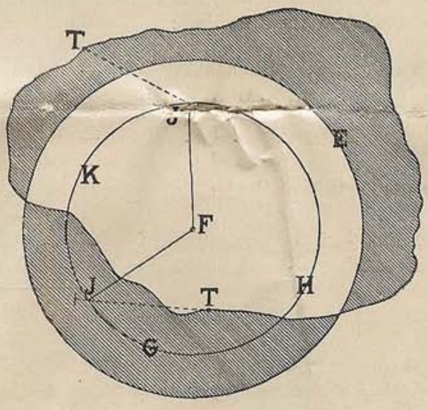


FIG. 3.

The area of the whole figure is equal, therefore, to the area of a rectangle whose side is the length of the movable arm J T, and whose base is equal to the length rolled off by the circumference of the wheel, *plus* the area of the circle E G T, whose radius is F T. As the various parts of different Planimeters are not all absolutely alike in length, constants have to be found for each one, such constants being the number of wheel units which would represent the area of the circle E G T. The value of a constant for one setting is given with each instrument. It will, of course, vary with different settings, as the revolutions of the wheel increase in proportion as the length of the arm J T is diminished, and vice-versa.

The constant may be easily ascertained by measuring a regular figure, such as a square or a circle, first with the fulcrum inside and then with the fulcrum outside. The difference of the two readings will be the constant. Care must be taken in the former case to note if the wheel record be positive or negative: this naturally depending upon the size of the figure.

Example: Find the constant for inside positions for a scale of 1″=5 feet, with a wheel unit of 0.4 sq. feet. To work this out, we draw a square $5'' \times 5'' = 25$ sq. inches. The scale of this drawing being assumed to be 1″=5 feet, or 1 sq. inch=25 sq. feet, it represents a square of $25 \times 25 = 625$ sq. feet, and each wheel unit being of the value of 0.4 sq. feet, the wheel would, if the fulcrum were outside the square, and the arm properly set, register $\frac{625}{.4} = 1562.5$ units while the tracer made the circuit of the square.

By Equations 2 and 3, $x = \frac{0.4}{25} = .016$ and $W = \frac{4.075}{.016} = 255$ units, we find that the movable arm of the particular instrument with which our examples have been worked out must be set at 9 in order that one circuit of the trial disc by the tracer shall register 255 units. Taking this setting, therefore, we put the weight with the fulcrum F inside the square, and placing the wheel and disc at zero, we proceed to take the tracer round the square; the whole of its perimeter being inside the circle described by the radius F T, the wheel consequently revolves backward and its record is negative. When the tracer has completed the circuit, the disc is found to have made nearly two whole revolutions, the exact reading of the wheel and disc being 3422; we have, therefore, the following:

First reading minus 20,000	=	− 20,000
Second reading . . .	=	+ 3,422
Difference . .	=	− 16,578

The constant for the inside position of the fulcrum is, therefore,

Reading with fulcrum outside, . . .	1562
Less reading with fulcrum inside,	− 16,578
	+ 18,140 = Constant.

The constant might also be found by taking a square $20'' \times 20'' = 400$ sq. inches, to the same scale of 1″ = 5 feet, or 1 sq. inch = 25 sq. feet, with a wheel unit of .4 sq. feet. The drawing would then represent a square of 10,000 square feet, and with this setting, $\frac{10,000}{.4} = 25,000$ wheel units.

With a drawing of this size, the tracer would always be outside the circumference of the circle whose radius is J T; consequently the wheel would continually revolve in a forward direction, and the final result registered would therefore be positive. Subtracting this result from 25,000 wheel units, we obtain the constant for the setting named.

In order to simplify the process of obtaining these constants, we furnish with the Planimeter when desired, at a small extra charge, an attachment by means of which the tracer can be rigidly connected with the fulcrum, so that the former travels automatically in a circle having the fulcrum for its centre. The area of this circle is easily ascertained by actual measurement, and the method of finding the constant carried out exactly as described.

As a rule, we do not advise inside positions of the fulcrum, as the results obtained by them are not generally as accurate as those rendered by outside positions. The better course is to divide the figure whose area is sought into parts sufficiently small to obtain their respective areas in the ordinary way with the fulcrum outside. As this process is however a lengthy one, recourse may be had to the former method when it is desired to find rapidly the contents of large areas.

We now give, for handy reference, a resumé of the different equations, throughout which the same letters represent the same denominations, all of which will be found in Fig. 1, namely

J F = The polar arm with its fulcrum F.

A T = The movable arm from zero to the tracer T.

J = The pivot or joint of the polar and movable arms.

B C D = The carriage with the slide for the movable arm.

W = The number of vernier units registered by the wheel (called wheel units) of which 1000 make one revolution of the wheel.

b = The total length of the movable arm from zero to the tracer.

a = The length of the movable arm from the carriage vernier zero to the joint J (a fixed quantity for each Planimeter).

c = The circumference of the wheel.

S = The length of the movable arm from zero to the carriage vernier zero= the setting or the adjustment.

M = The length of the movable arm from the joint J to the tracer T.

$b-a$ = The sum of the two variable sections of the movable arm, whose length is always M+S.

D = The scale of the drawing representing the figure whose area is desired.

x = The absolute value of the wheel unit, or its ratio to a scale of 1=1.

y = The comparative value of the wheel unit, or its ratio to the scale of the drawing.

$$y = D^2 \times x \quad \text{Eq. 1.}$$

$$x = \frac{y}{D^2} \quad \text{Eq. 2.}$$

$$W = \frac{\text{Area of trial disc}}{x} \quad \text{Eq. 3.}$$

$$y = \frac{D^2 \times \text{Area of trial disc}}{W} \quad \text{Eq 4.}$$

$$D^2 = \frac{y \times W}{\text{Area of trial disc}} \quad \text{Eq. 5.}$$

$$\text{Area of any figure} = M \times \frac{c\,W}{1000} \quad \text{Eq. 6.}$$

$$M = b - a - S \quad \text{Eq. 7.}$$

$$S = b - a - M \quad \text{Eq. 8.}$$

$$\text{Area of any figure} = (b - a - S) \times \frac{c\,W}{1000} \quad \text{Eq. 9.}$$

$$\text{Area for 1 revolution of wheel} = (b - a - S) \times c \quad \text{Eq. 10.}$$

$$c = \frac{\text{Area}}{b - a - S} \quad \text{Eq. 11.}$$

$$S = b - a - \frac{\text{Area}}{c} \quad \text{Eq. 12.}$$

The following tables give a list of the different scales usually employed for working drawings of machinery, buildings, etc., plans of surveys, railways, canals, etc., with the absolute value x of the wheel unit; the value y assigned to the wheel unit as compared with the scale of the drawing; the number of wheel units registered by one complete circuit of the trial disc, for areas of 2570 to 2630 square millimeters, or for their equivalents 3.98½ to 4.07½ square inches; also a list of approximate settings S, which will materially assist in finding the exact setting for any scale.

No.	Scale of Drawing. Ratio.	Scale of Drawing. Lineal Scale. D	Scale of Drawing. Surface Scale. D^2	Absolute Value of Wheel Unit. Scale 1=1. x	Comparative Value of Wheel Unit. Scale of Drawing. y	Number of Wheel Units registered for One Complete Circuit of the Trial Disc. □ M/M. 2570 sq. in. 3.98½	□ M/M. 2580 sq. in 4.00	□ M/M. 2590 sq. in 4.01½	□ M/M. 2600 sq. in. 4.03	□ M/M. 2610 sq. in 4.04½	□ M/M. 2620 sq. in. 4.06	□ M/M. 2630 sq. in. 4.07½	Approximate Setting. S
		IN. FT.	SQ. IN. SQ. FT.	SQ. FT.	SQ. FT.								
1	1 : 2	6 =1	1=0.0278	.0072	.0002	554	556	558	560	562	564	566	196
2	1 : 4	3 =1	1=0.1111	.0009	.001	443	444½	446	447	449	451	453	156
3	1 : 8	1½=1	1=0.4444	.0009	.004	443	444½	446	447	449	451	453	156
4	1 : 16	¾=1	1=1.777	.01125	.02	354	355½	357	358	359	361	362	108
5	1 : 32	⅜=1	1=7.111	.00703	.05	567	569	571	573	575	577	579	198
6	1 : 12	1 =1	1= 1	.01	.01	398½	400	401½	403	404½	406	407½	137
7	1 : 24	1 =2	1= 4	.0125	.05	319	320	321	322	324	325	326	84
8	1 : 48	1 =4	1= 16	.015625	.25	255	256	257	258	259	260	261	18
9	1 : 60	1 =5	1= 25	.008	.2	498	500	502	504	506	508	510	175
10	1 : 96	1 =8	1= 64	.078125	.5	510	512	514	516	518	520	522	182
11	1 : 120	1 =10	1= 100	.01	1.0	398½	400	401½	403	404½	406	407½	137
12	1 : 192	1 =16	1= 256	.078125	2	510	512	514	516	518	520	522	182
13	1 : 240	1 =20	1= 400	.01	4	398½	400	401½	403	404½	406	407½	137
14	1 : 300	1 =25	1= 625	.016	10	249	250	251	252	253	254	255	11
15	1 : 360	1 =30	1= 900	.00888	8	449	451	452	453	455	457	458	160
16	1 : 480	1 =40	1= 1,600	.0125	20	319	320	321	322	324	325	326	84
17	1 : 600	1 =50	1= 2,500	.01	25	398½	400	401½	403	404½	406	407½	137
18	1 : 1200	1 =100	1=10,000	.01	100	398½	400	401½	403	404½	406	407½	137
		IN. IN.	SQ. IN. SQ IN.	SQ. IN.	SQ. IN.								
19	1 : 1	1 =1	1= 1	.01	.01	398½	400	401½	403	404½	406	407½	137
20	1 : 10	1 =10	1= 100	.01	1	398½	400	401½	403	404½	406	407½	137
21	1 : 100	1 =100	1= 10,000	.01	100	398½	400	401½	403	404½	406	407½	137
22	1 : 1000	1 =1000	1=1,000,000	.01	10,000	398½	400	401½	403	404½	406	407½	137
		IN. MILE.	SQ. IN.	SQ. CHAINS.	SQ. CHAINS.								
23	1 : 528	120 =1	1= $\frac{4}{9}$ sq. chains.	.01125	.005	354	355½	357	358	359	361	362	108
24	1 : 792	80 =1	1= 1 "	.01	.01	398½	400	401½	403	404½	406	407½	137

25	1 : 1056	60 =1	1= 1 7/9 "	.01125	.02	354	355½	357	358	359	361	362	108
26	1 : 1584	40 =1	1= 4 "	.0125	.05	319	320	321	322	324	325	326	84
27	1 : 1980	32 =1	1= 6¼ "	.008	.05	498	500	502	504	506	508	510	175
28	1 : 3168	20 =1	1=16 "	.015625	.25	255	256	257	258	259	260	261	18
29	1 : 3960	16 =1	1=25 "	.008	.2	498	500	502	504	506	508	510	175
30	1 : 5280	12 =1	1=14 4/9 "	.01125	.5	354	355½	357	358	359	361	362	108
31	1 : 6336	10 =1	1= { 64 sq. chains. .01 sq. miles. }	.078125	.5	510	512	514	516	518	520	522	182
32	1 : 7920	8 =1	1= { 100 sq. chains. 10 acres. }	.01	1	398½	400	401½	403	404½	406	407½	137
33	1 : 10560	6 =1	1=177 7/9 sq. chains.	.01125	2	354	355½	357	358	359	361	362	108
34	1 : 12672	5 =1	1= { 256 sq. chains. .04 sq. miles. }	.01	4	398½	400	401½	403	404½	406	407½	137
35	1 : 15840	4 =1	1= { 400 sq. chains. 40 acres. }	.0125	5	319	320	321	322	324	325	326	84
36	1 : 21120	3 =1	1=711 1/9 sq. chains.	.00844	6	472	474	476	477	479	481	483	168
				SQ. MILES.	SQ. MILES.								
37	1 : 25344	2½=1	1=.16 sq. miles.	.015625	.25	255	256	257	258	259	260	261	18
		M/M. M/M.	SQ. M/M. SQ. M/M.	SQ. M/M.	SQ. M/M.								
38	1 : 1	1 =1	1= 1	5	5	514	516	518	520	522	524	526	182
39	1 : 2	1 =2	1= 4	5	20	514	516	518	520	522	524	526	182
40	1 : 5	1 =5	1= 25	4	100	642½	645	647½	650	652½	655	657½	215
41	1 : 10	1 =10	1= 100	5	500	514	516	518	520	522	524	526	182
					SQ. METRES.								
42	1 : 20	1 =20	1= 400	5	.002	514	516	518	520	522	524	526	182
43	1 : 25	1 =25	1= 625	8	.005	321	323	324	325	326	327½	329	85
44	1 : 50	1 =50	1= 2,500	4	.01	642½	645	647½	650	652½	655	657½	215
			SQ. METRES.										
45	1 : 100	1 =100	1= .01	5	.05	514	516	518	520	522	524	526	182
46	1 : 250	1 =250	1= .0625	8	.5	321	323	324	325	326	327½	329	85
47	1 : 500	1 =500	1= .25	4	.1	642½	645	647½	650	652½	655	657½	215
		METRES.											
48	1 : 1000	1 =1	1= 1	5	5	514	516	518	520	522	524	526	182
49	1 : 1250	1 =1¼	1= 1.5625	9.6	15	268	269	270	271	272	273	274	30
50	1 : 2500	1 =2½	1= 6.25	8	50	321	323	324	325	326	327½	329	85
51	1 : 4000	1 =4	1= 16	6.25	100	411	413	414	416	418	419	421	140
52	1 : 5000	1 =5	1= 25	4	100	642½	645	647½	650	652½	655	657½	215
53	1 : 10000	1 =10	1=100	5	500	514	516	518	520	522	524	526	182
54	1 : 20000	1 =20	1=400	5	2000	514	516	518	520	522	524	526	182

In the following examples we assume our trial disc to be 2630 sq. m/m, or 4.07½ sq. inches.

Example: Required the area of a plane figure, such as an indicator diagram, made out to a scale of $1'' = 1''$.

We assign, according to No. 19, the value of .01 sq. inch to each wheel unit, and we find that the wheel should register 407½ units for a complete circuit of the trial disc, and also that the setting should be about 137. By trial we find that a setting of 135,6 gives the required number (407½) of units. We therefore circumscribe the figure with the tracer, and find that the wheel has registered say 247 units. The exact area of the figure is, therefore, $247 \times .01 = 2.47$ sq. inches. Assuming that this is an engine indicator diagram, and that the distance between the terminal points of the diagram is 3.78 inches, and that a spring No. 40 has been used, we obtain the mean effective pressure by dividing 40 by 3.78 and multiplying the quotient by 2.47, thus:

$$\frac{40 \times 2.47}{3.78} = 26.13 \text{ lbs. per square inch.}$$

Example: Required the area of a pond shown on a plan to a scale of 40 inches to a mile.

By No. 26 we assign a value of .05 sq. chains to each wheel unit, and we find that the wheel should register 326 units for one circuit of the trial disc, and that the setting should be about 84.5. Having found the exact setting as in the previous case, and taken due note of the reading of the wheel and disc, say 3856, we then circumscribe the figure with the tracer, and now find the wheel and disc stand at 5109. We then have

Second reading, ...	= 5109
First "	= 3856
Difference, ...	= 1253

then the area of the pond $= 1253 \times .05 = 62.65$ sq. chains $= 6.265$ sq. acres.

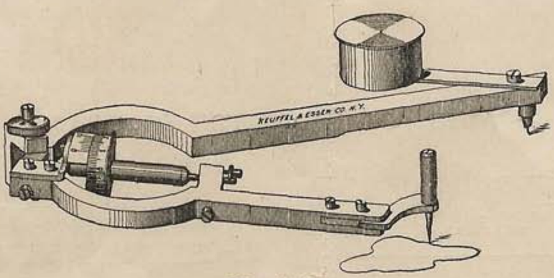
Example: Required the area of displacement of a floating body shown on a drawing of 1 : 100, or 1 m/m = 100 m/m, and 1 sq. m/m = .01 sq. meters.

By No. 45 each wheel unit would represent an area of .05 sq. meters, and for a complete circuit of the trial disc 526 units should be registered. The exact setting being found by means of the approximative one of 182, and the reading of the wheel and disc, say 8655, having been noted, we pass the tracer round the object, and then find the reading to be 1342. We have, therefore,

Second reading + 10,000	= 11,342
First "	= 8,655
Difference, ...	= 2,687

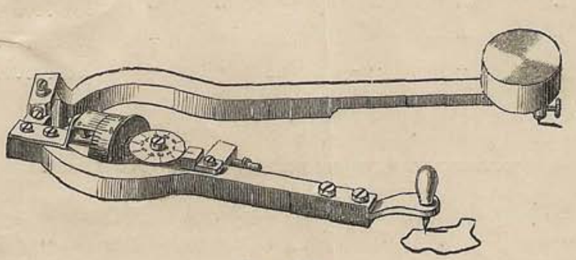
The area of displacement is, therefore, $2687 \times .05 = 134.35$ square meters.

We have, also, smaller and simpler Planimeters, which, however, are not adapted to general use. They are:



No. 1110.

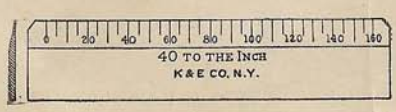
1110. Which measures only square inches. The arm is a fixed one, its length being so adjusted to the diameter of the wheel and the position of the joint that one complete revolution of the wheel indicates an actual area of 10 square inches (that is, to a scale of $1 = 1$, or to the scale of the drawing). With the aid of the vernier, inches, tenths and hundredths of an inch may be read off. Of course any *comparative* value y may be assigned to the wheel unit if the drawing be to a scale of 1 : 1, 1 : 10, 1 : 100, etc. This Planimeter is specially designed for obtaining the areas of Steam Engine Indicator Diagrams.



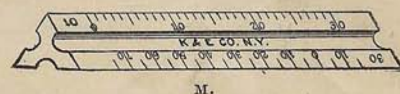
No. 1111.

1111. Is a similar instrument to No. 1110, with however the addition of a horizontal disc, which permits of any number of revolutions of the wheel up to 10 being noted. It may be used for any small drawings made to scales 1 : 1, 1 : 10, 1 : 100, etc.

SCALES FOR INDICATOR DIAGRAMS.



C.



M.

U. S. STANDARD.

FLAT BOXWOOD SCALES, ONE EDGE BEVELED AND DIVIDED.

A.—Indicator Scale,	4 inch,	graduated to	10	parts to	the inch	....	each	$0.30
B.—	"	"	20	"	"	....	"	.30
C.—	"	"	40	"	"	....	"	.30
D.—	"	"	50	"	"	....	"	.30
E.—	"	"	60	"	"	....	"	.30
F.—	"	"	80	"	"	....	"	.40
G.—	"	"	100	"	"	....	"	.40
H.—	"	"	12	"	"	....	"	.30
I.—	"	"	24	"	"	....	"	.30
K.—	"	"	32	"	"	....	"	.30
L.—	"	"	64	"	"	....	"	.30

TRIANGULAR BOXWOOD SCALES, SIX DIVIDED EDGES.

M.—Indicator Scales,	3 inch,	graduated	10, 20, 30, 40, 50, 60	parts to inch,	each,	$0.75
N.—	"	"	20, 40, 50, 60, 80, 100	"	"	.75
O.—	"	"	10, 15, 25, 30, 40, 70	"	"	.75
P.—	"	"	10, 20, 25, 60, 80, 100	"	"	.75

Scales with different graduations made to order, in reasonable quantities, at proportionate prices.