Math 20E Homework Assignment 2
Due 11:00pm Thursday, February 2, 2023

1. Change the order integration and evaluate:

$$\int_{y=0}^{1} \int_{x=y}^{1} \sin\left(x^2\right) \, dx \, dy.$$

2. Change the order integration and evaluate:

$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} e^{x^3} \, dx \, dy.$$

3. Evaluate the integral $\iiint_W z \, dx \, dy \, dz$; where W is the region bounded by $x = 0$, $y = 0$, $z = 0$, $z = 1$, and the cylinder $x^2 + y^2 = 1$, with $x \geq 0$, $y \geq 0$.

4. Let D^* be the parallelogram with vertices at $(-1, 3)$, $(0, 0)$, $(2, -1)$, and $(1, 2)$. Let D be the rectangle $D = [0, 1] \times [0, 1]$. Find a T such that D is the image set of D^* under T; that is, $D = T(D^*)$.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the spherical coordinate mapping defined by $(\rho, \phi, \theta) \mapsto (x, y, z)$, where

$$x = \rho \sin(\phi) \cos(\theta), \qquad y = \rho \sin(\phi) \sin(\theta), \qquad z = \rho \cos(\phi).$$

Let D^* be the set of points (ρ, ϕ, θ) such that $\rho \in [0, 1]$, $\phi \in [0, \pi]$, $\theta \in [0, 2\pi]$.

   (a) Find $D = T(D^*)$.

   (b) Is T one-to-one? If not, can we eliminate a subset $S \subseteq D^*$ so that T is one-to-one on the remainder $D^* \setminus S = \{(x, y, z) \in D^* \mid (x, y, z) \notin S\}$?

6. Evaluate $\iint_D x^2 \, dx \, dy$ where D is determined by the two conditions $0 \leq x \leq y$ and $x^2 + y^2 \leq 1$.

7. Evaluate $\iiint_W \sqrt{x^2 + y^2 + z^2} \, e^{-(x^2+y^2+z^2)} \, dx \, dy \, dz$, where W is the solid bounded by the two spheres

   $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ with $0 < a < b$.

8. Evaluate $\iint_R (x+y) \, dx \, dy$, where R is the rectangle in the xy-plane with vertices at $(0, 1), (1, 0), (3, 4), (4, 3)$.

9. Show that the path $\mathbf{c}(t) = \left(\sin(t), \cos(t), e^t\right)$ is a flow line of the vector field $\mathbf{F}(x, y, z) = (y, -x, z)$.

10. Let $\mathbf{F}(x, y, z) = (yz, xz, xy)$. Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\mathbf{F} = \nabla f$.