Math 20E

September 5, 2017

Question 1 Suppose $\mathbf{F}$ is a C^1 vector field on $\mathbb{R}^3$. Let H be the unit hemisphere given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$, let D be the unit disk given by $z = 0$ with $x^2 + y^2 \leq 1$, and let $S = H \cup D$. Then,

A. $\partial H = \partial D$, including orientation when H and D are both oriented with the upward-pointing unit normal vector.

B. $\iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

C. $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} + \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

D. $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0$.

*__E.__ **A**, **B** and **D**

Question 2 Given W a symmetric elementary solid region and ∂W its positively oriented bounding surface. If $\mathbf{F}$ is a C^1 vector field defined on $W \cup \partial W$, then

A. $\displaystyle\iiint_W \nabla \cdot \mathbf{F}\, dV = \iint_{\partial W} \mathbf{F} \cdot d\mathbf{S}$.

B. The boundary surface ∂W is positively oriented with respect to the orientation of the solid region W given by the right-hand rule.

C. There is no orientation for the boundary surface ∂W because the solid region W cannot be oriented.

***D.** **A** and **B**.

E. **A** and **C**.

Question 3 Suppose $\mathbf{F}$ is a C^2 vector field on $\mathbb{R}^3$, and let S be the unit sphere given by $x^2 + y^2 + z^2 = 1$. Then, $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. by Stokes' Theorem is equal to $\displaystyle\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

B. by Gauss's Theorem is equal to $\displaystyle\iiint_B \nabla \cdot (\nabla \times \mathbf{F}) dV$,

where B is the unit ball given by $x^2 + y^2 + z^2 \leq 1$.

C. is 0.

D. **A** and **B**.

***E.** **A**, **B**, and **C**.

Question 4 Suppose $\mathbf{F}$ is a C^1 vector field in $\mathbb{R}^3$ and that W is a solid region in $\mathbb{R}^3$ that can be decomposed into finitely many symmetric elementary regions with a positively oriented boundary surface ∂W.

$\iint_{\partial W} \mathbf{F} \cdot d\mathbf{S}$ is called the flux of $\mathbf{F}$ across the surface ∂W.

Then,

A. $\iint_{\partial W} \mathbf{F} \cdot d\mathbf{S} = \iiint_W \nabla \cdot \mathbf{F} \, dV$.

B. the units of $\nabla \cdot \mathbf{F}$ are flux per unit volume.

C. the average value of $\nabla \cdot \mathbf{F}$ over W is $\dfrac{\iint_{\partial W} \mathbf{F} \cdot d\mathbf{S}}{\iiint_W dV}$.

D. **A** and **B**.

***E.** **A**, **B**, and **C**.

Question 5 Give a continuous vector field $\mathbf{F}$ in $\mathbb{R}^3$, and let $\mathbf{c} : [a, b] \to C$ be a regular C^1 path in $\mathbb{R}^3$ with image curve $C \subset \mathbb{R}^3$. Then, $\tau(t) = \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is a continuous unit tangent vector along C and

A. $\dfrac{\int_C (\mathbf{F} \cdot \boldsymbol{\tau})\, ds}{\int_C ds}$ is the average value of the tangential component of $\mathbf{F}$ along C.

B. The average value of the tangential component of $\mathbf{F}$ along C is independent of the path $\mathbf{c}$ chosen to parametrize C.

C. The average value of the tangential component of $\mathbf{F}$ along C would change sign if a path γ with opposite orientation than $\mathbf{c}$ were chosen to parametrize C.

D. **A** and **B**.

***E.** **A** and **C**.

Question 6 Given $\mathbf{F}$ a continuous vector field defined on a regular C^1 parametrized surface $\Phi : D \to S$ with image surface $S \subset \mathbb{R}^3$. Then

$$\mathbf{n} = \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}$$

is a continuous unit normal vector on S. The average value of the normal component of $\mathbf{F}$ over S

A. is given by $\dfrac{\iint_\Phi \mathbf{F} \cdot d\mathbf{S}}{\iint_\Phi dS}$.

B. is independent of the choice of parametrized surface Φ used to parametrize S.

C. has opposite sign on a parametrized surface Ψ with opposite orientation from Φ.

D. **A** and **B**.

***E.** **A** and **C**.