Math 20E

September 7, 2017

Question 1 The derivative of a function $f(x, y)$ at a point (a, b) is a

A. number.

B. the slope of the tangent plane at (a, b).

C. the matrix of partial derivatives of f at (a, b).

D. the gradient of f at (a, b).

*__E.__ both **C** and **D**.

Question 2 The speed of an object is constant. The object's

***A.** velocity and acceleration are orthogonal.

B. acceleration is zero.

C. velocity is constant.

D. both **B** and **C**.

E. None of the above.

Question 3 Given domains $D \subset \mathbb{R}^2$ and $S \subset \mathbb{R}^2$ and a one-to-one transformation $T : D \to S$ that maps D onto S. Then T can be used to change variables as follows:

A. $$\iint_S f(x,y)\,dx\,dy = \iint_D f\left(T\left(u,v\right)\right)\left|\text{det}\left[\mathbf{D}T\left(u,v\right)\right]\right|\,du\,dv.$$

B. $$\iint_D f(u,v)\,du\,dv = \iint_S f\left(T\left(x,y\right)\right)\left|\text{det}\left[\mathbf{D}T\left(x,y\right)\right]\right|\,dx\,dy.$$

C. $$\iint_D f(u,v)\,du\,dv = \iint_S f\left(T^{-1}\left(x,y\right)\right)\left|\text{det}\left[\mathbf{D}T^{-1}\left(x,y\right)\right]\right|\,dx\,dy.$$

D. Both **A** and **B**.

***E.** Both **A** and **C**.

Question 4 Given a path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is *regular* at t_0 means

A. the derivative $\mathbf{c}'(t_0)$ exists.

B. the derivative $\mathbf{c}'(t_0)$ exists and is not zero.

C. the image curve $\mathbf{c}([a, b])$ has a tangent vector at $\mathbf{c}(t_0)$.

D. $\mathbf{c}'(t_0)$ is a unit vector.

*__E.__ both **B** and **C**; they are equivalent.

Question 5 Given $f : \mathbb{R}^n \to \mathbb{R}$ a C^1 scalar-valued function and $\mathbf{c} : [a, b] \to \mathbb{R}^n$ a simple C^1 path,

A. $\mathbf{D}f(t) = \nabla f\left(\mathbf{c}\left(t\right)\right) \cdot \mathbf{c}'(t)$ by the chain rule.

B. $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = \int_a^b \nabla f\left(\mathbf{c}\left(t\right)\right) \cdot \mathbf{c}'(t)\, dt.$

C. The value of $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s}$ is independent of the path $\mathbf{c}$.

D. $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = f\left(\mathbf{c}\left(b\right)\right) - f\left(\mathbf{c}\left(a\right)\right)$, which depends only on the value of f at the endpoints $\mathbf{c}(a)$ and $\mathbf{c}(b)$.

***E.** All of the above.

Question 6 Given a parametrized surface $\Phi : D \to \mathbb{R}^3$. Φ is *regular* at (u_0, v_0) means

*__A.__ the vector $\mathbf{T}_u \times \mathbf{T}_v$ is not zero at (u_0, v_0).

B. the vector $\mathbf{T}_u \times \mathbf{T}_v$ is normal to the surface $S = \Phi(D)$ at (u_0, v_0).

C. the surface $S = \Phi(D)$ has a tangent plane at $\Phi(u_0, v_0)$.

D. $\mathbf{T}_u \times \mathbf{T}_v$ at (u_0, v_0) is a unit vector.

E. **A**, **B**, and **C**.

Question 7 Given a continuous vector field $\mathbf{F}$ on $\mathbb{R}^n$ and a regular C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. Then,

A. The unit tangent vector $\boldsymbol{\tau}(t) = \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is defined at each point along the path $\mathbf{c}$.

B. The line integral of $\mathbf{F}$ along $\mathbf{c}$ is given by
$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\, dt.$$

C. The path integral of the tangential component of $\mathbf{F}$ along $\mathbf{c}$ is given by
$$\int_{\mathbf{c}} (\mathbf{F} \cdot \boldsymbol{\tau})\, ds = \int_a^b \left(\mathbf{F}(\mathbf{c}(t)) \cdot \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|} \right) \|\mathbf{c}'(t)\|\, dt.$$

D. $\displaystyle\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \int_{\mathbf{c}} (\mathbf{F} \cdot \boldsymbol{\tau})\, ds.$

***E.** All of the above.

Question 8 Given $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ a continuous vector field in $\mathbb{R}^3$, with $\Phi : D \to S$ a regular parametrized surface in $\mathbb{R}^3$. Then, $\mathbf{n} = \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}$ is defined at each point $\Phi(u, v)$ on the surface S and it follows that

A. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi\left(u, v\right)\right) \cdot \left(\mathbf{T}_u \times \mathbf{T}_v\right)\, du\, dv.$

B. $\displaystyle\iint_\Phi \left(\mathbf{F} \cdot \mathbf{n}\right)\, dS =$
$$\iint_D \left(\mathbf{F}\left(\Phi\left(u, v\right)\right) \cdot \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}\right) \|\mathbf{T}_u \times \mathbf{T}_v\|\, du\, dv.$$

C. $\displaystyle\iint_D \mathbf{F}\left(\Phi\left(u, v\right)\right) \cdot \left(\mathbf{T}_u \times \mathbf{T}_v\right)\, du\, dv =$
$$\iint_D \left(\mathbf{F}\left(\Phi\left(u, v\right)\right) \cdot \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}\right) \|\mathbf{T}_u \times \mathbf{T}_v\|\, du\, dv.$$

D. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_\Phi \left(\mathbf{F} \cdot \mathbf{n}\right)\, dS.$

*__E.__ All of the above.

Question 9 Let $S \subset \mathbb{R}^3$ be a surface with regular C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))\,.$$

A. $\mathbf{T}_u = \frac{\partial \Phi}{\partial u}$ and $\mathbf{T}_v = \frac{\partial \Phi}{\partial v}$ are vectors tangent to S.

B. $\mathbf{T}_u \times \mathbf{T}_v$ is a vector normal to S.

C. The area of $\Phi([u, u + \Delta u] \times [v, v + \Delta v])$ on S is approximately $||\mathbf{T}_u(u,v) \times \mathbf{T}_u(u,v)||\, \Delta u\, \Delta v$.

D. **A** and **B**.

***E.** **A**, **B** and **C**.

Question 10 Given a surface S with two distinct C^1 parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, then

***A.** $\displaystyle\iint_\Phi f\, dS = \iint_\Psi f\, dS.$

B. $\displaystyle\iint_\Phi f\, dS < \iint_\Psi f\, dS$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$.

C. $\displaystyle\iint_\Phi f\, dS = -\iint_\Psi f\, dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = -\left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\right)$.

D. **B** and **C**.

E. none of the above.

Question 11 Given a surface S with two distinct C^1 parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_\Psi \mathbf{F} \cdot d\mathbf{S}$.

B. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_\Psi \mathbf{F} \cdot d\mathbf{S}$ when $\displaystyle\frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}||} = \frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||}$.

C. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} = -\iint_\Psi \mathbf{F} \cdot d\mathbf{S}$ when $\displaystyle\frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}||} = -\frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||}$.

D. $\displaystyle\iint_\Phi \mathbf{F} \cdot d\mathbf{S} < \iint_\Psi \mathbf{F} \cdot d\mathbf{S}$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$.

*__E.__ **B** and **C**.

Question 12 Given a simple domain D with C^1 boundary ∂D, the area of D is given by

A. $A(D) = \iint_D dx\,dy.$

B. $A(D) = -\int_{\partial D} y\,dx.$

C. $A(D) = \dfrac{1}{2}\int_{\partial D} x\,dy - y\,dx.$

D. **A** and **C**.

*__E.__ **A**, **B** and **C**.

Question 13 Given an orientable surface S with boundary curve C, and a C_1 vector field $\mathbf{F}$. Then,

A. $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{s}$.

B. Given a path $\mathbf{c}$ that parametrizes the curve C,
$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \pm \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, depending on the orientation chosen for S.

C. Given a parametrization $\Phi : D \to S$,

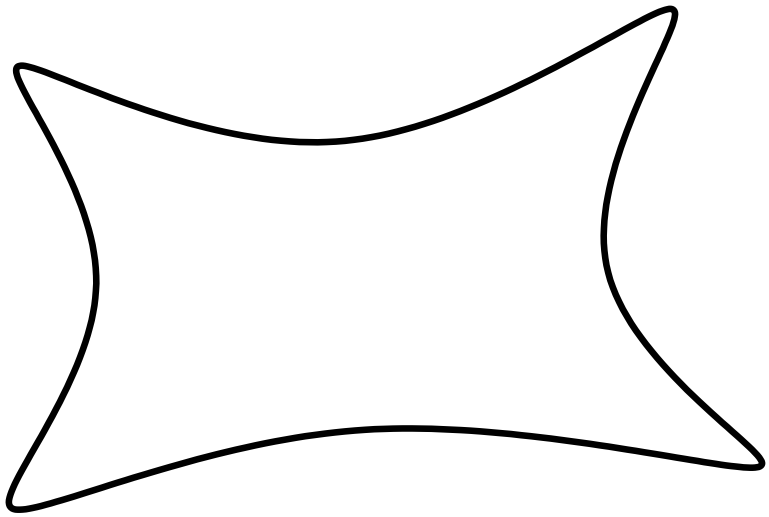
$$\iint_\Phi \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_{\partial\Phi} \mathbf{F} \cdot d\mathbf{s},$$

where $\partial\Phi$ is the positively oriented boundary curve with respect to the orientation of Φ.

D. **A** and **C**.

***E.** **B** and **C**.

Question 14 Consider the following domain D:



A. By Green's theorem, the area of D may be computed by evaluating $\frac{1}{2}\int_{\partial D} x\,dy - y\,dx$.

B. Green's theorem cannot be applied on D since D is not a simple region.

C. The area of D could be measured by tracing ∂D with a planimeter.

***D.** **A** and **C**.

E. **B** and **C**.

Question 15 Suppose $\mathbf{F}$ is a C^1 vector field on the unit sphere S in $\mathbb{R}^3$. Then, $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. is 0.

B. is most easily computed by parametrizing S using latitude-longitude coordinates.

C. is most easily computed by applying Stokes' theorem and computing $\displaystyle\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

D. cannot be computed using Stokes' theorem because the sphere S has no boundary curve ∂S.

***E.** **A** and **C**: the line integral in **C** is 0 because the boundary curve ∂S is empty.

Question 16 Suppose $\mathbf{F}$ is a C^1 vector field on $\mathbb{R}^3$. Let H be the unit hemisphere given by $x^2+y^2+z^2=1$ with $z \geq 0$, let D be the unit disk given by $z=0$ with $x^2+y^2 \leq 1$, and let $S = H \cup D$. Then,

A. $\partial H = \partial D$, including orientation when H and D are both oriented with the upward-pointing unit normal vector.

B. $\displaystyle\iint_H (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \iint_D (\nabla\times\mathbf{F})\cdot d\mathbf{S}$.

C. $\displaystyle\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \iint_H (\nabla\times\mathbf{F})\cdot d\mathbf{S} + \iint_D (\nabla\times\mathbf{F})\cdot d\mathbf{S}$.

D. $\displaystyle\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S} = 0$.

***E.** **A**, **B** and **D**.

Question 17 Suppose $\mathbf{F}$ is a C^2 vector field on $\mathbb{R}^3$, and let S be the unit sphere given by $x^2 + y^2 + z^2 = 1$. Then, $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. by Stokes' Theorem is equal to $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

B. by Gauss's Theorem is equal to $\iiint_B \nabla \cdot (\nabla \times \mathbf{F}) dV$, where B is the unit ball given by $x^2 + y^2 + z^2 \leq 1$.

C. is 0.

D. **A** and **B**.

*__E.__ **A**, **B**, and **C**.

Question 18 Let $\mathcal{P}$ be a solid region that looks a little like a potato placed in $\mathbb{R}^3$ so that the origin $(0,0,0)$ is somewhere inside the potato. It's bounding surface $\partial\mathcal{P}$ looks a little like a potato skin and is oriented with the outward-pointing unit normal. Then,

*__A.__ $\displaystyle\iint_{\partial\mathcal{P}} \frac{\mathbf{r}}{r^3} \cdot d\mathbf{S} = 4\pi.$

B. $\displaystyle\iint_{\partial\mathcal{P}} \frac{\mathbf{r}}{r^3} \cdot d\mathbf{S} = 0.$

C. $\displaystyle\iint_{\partial\mathcal{P}} \frac{\mathbf{r}}{r^3} \cdot d\mathbf{S} = \iiint_{\mathcal{P}} \boldsymbol{\nabla} \cdot \frac{\mathbf{r}}{r^3}\, dV$, by Gauss's Theorem.

D. **A** and **C**.

E. **B** and **C**.

Question 19 The use of clickers in this course was

A. very helpful for reviewing the important conceptual ideas of the subject.

B. an easy way to earn extra credit and how the professor encouraged me to get to class on time.

C. a fun way to start each class period.

***D.** All of the above. Clickers get everyone involved.

E. None of the above. Clickers are a complete waste of time.