Math 20E

August 14, 2017

Question 1 Given a C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is a regular path means

A. its image curve $C = \mathbf{c}([a, b])$ has a tangent vector at every point.

B. $\mathbf{c}'(t) \neq \mathbf{0}$ for every $t \in [a, b]$.

C. the path $\mathbf{c}$ is ordinary.

***D.** **A** and **B**.

E. none of the above; in fact, regular paths are quite extraordinary.

Question 2 Recall that two vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ are orthogonal if and only if $\mathbf{v} \cdot \mathbf{w} = 0$. Which of the following statements are true?

A. If $\mathbf{v}$ and $\mathbf{w}$ are orthogonal, then the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\frac{\pi}{2}$.

B. If $\mathbf{c} : [a, b] \to \mathbb{R}^n$ is a C^1 path, then $\mathbf{c}'(t)$ is a tangent vector to $\mathbf{c}(t)$ at every $t \in [a, b]$.

C. $\mathbf{0}$ is the only vector in $\mathbb{R}^n$ that is orthogonal to itself.

D. **B** is true when $\mathbf{c}$ is a regular path.

***E.** **C** and **D**.

Question 3 The paths

$$\begin{array}{lcl} \mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^2 & & \mathbf{c}_2 : \left[0, \sqrt{2\pi}\right] \to \mathbb{R}^2 \\ \mathbf{c}_1(t) = (\cos(t), \sin(t)) & \text{and} & \mathbf{c}_2(t) = \left(\cos\left(t^2\right), \sin\left(t^2\right)\right) \end{array}$$

have the same image curve C; that is,

$$C = \mathbf{c}_1([0, 2\pi]) = \mathbf{c}_2\left(\left[0, \sqrt{2\pi}\right]\right).$$

We can conclude that

A. the curve C has a well-defined tangent line at every point.

B. $\mathbf{c}_1$ is a regular path.

C. $\mathbf{c}_2$ is not a regular path.

D. **A** and **B**

*__E.__ **A**, **B**, and **C**

Question 4 In order for a transformation $T : R \to S$ to be a coordinate transformation that can be used to change variables in a double or triple integral, it should

A. be a one-to-one mapping mapping of R

B. map R onto S

C. both **A** and **B**

*__D.__ both **A** and **B**, except that would be OK if it failed to be one-to-one on parts of the boundary of R

E. none of the above: "one-to-one" and "onto" have nothing to do with coordinate transformations.

Question 5 Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. Then, $T(\mathbf{x}) = A\,\mathbf{x}$ for some 2×2 matrix A. Suppose also that $\det(A) \neq 0$. Then,

A. if P is a parallelogram, so is $T(P)$.

B. if $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are the vertices of a triangle $\triangle$, then $T(\mathbf{x}_1)$, $T(\mathbf{x}_2)$, and $T(\mathbf{x}_3)$ are the vertices of $T(\triangle)$.

C. if C is a circle, then so is $T(C)$.

*__D.__ **A** and **B**

E. **A**, **B**, and **C**