Math 20E

August 15, 2017

Question 1 The paths

$$\begin{array}{l} \mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^2 \\ \mathbf{c}_1(t) = (\cos(t), \sin(t)) \end{array} \quad \text{and} \quad \begin{array}{l} \mathbf{c}_2 : \left[0, \sqrt{2\pi}\right] \to \mathbb{R}^2 \\ \mathbf{c}_2(t) = \left(\cos\left(t^2\right), \sin\left(t^2\right)\right) \end{array}$$

have the same image curve C; that is,

$$C = \mathbf{c}_1([0, 2\pi]) = \mathbf{c}_2\left(\left[0, \sqrt{2\pi}\right]\right).$$

We can conclude that

A. the curve C has a well-defined tangent line at every point.

B. $\mathbf{c}_1$ is a regular path.

C. $\mathbf{c}_2$ is not a regular path.

D. **A** and **B**

***E.** **A**, **B**, and **C**

Question 2 Given a C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is a regular path means

A. its image curve $C = \mathbf{c}\,([a, b])$ has a tangent vector at every point.

B. $\mathbf{c}'(t) \neq \mathbf{0}$ for every $t \in [a, b]$.

C. the path $\mathbf{c}$ is ordinary.

***D.** **A** and **B**.

E. none of the above; in fact, regular paths are quite extraordinary.

Question 3 In order for a transformation $T : R \to S$ to be a coordinate transformation that can be used to change variables in a triple integral, it should

A. be a one-to-one mapping mapping of the solid region R.

B. map the solid region R onto the solid region S.

C. both **A** and **B**.

***D.** both **A** and **B**, except that would be OK if it failed to be one-to-one on parts of the boundary surface of R.

E. none of the above: "one-to-one" and "onto" have nothing to do with coordinate transformations.

Question 4 Given domains $D \subset \mathbb{R}^2$ and $S \subset \mathbb{R}^2$ and a one-to-one transformation $T : D \to S$ that maps D onto S. Then T can be used to change variables as follows:

A. $\displaystyle\iint_S f(x,y)\,dx\,dy = \iint_D f\left(T(u,v)\right)\,\left|\det\left[\mathbf{D}T(u,v)\right]\right|\,du\,dv.$

B. $\displaystyle\iint_D f(u,v)\,du\,dv = \iint_S f\left(T(x,y)\right)\,\left|\det\left[\mathbf{D}T(x,y)\right]\right|\,dx\,dy.$

C. $\displaystyle\iint_D f(u,v)\,du\,dv = \iint_S f\left(T^{-1}(x,y)\right)\,\left|\det\left[\mathbf{D}T^{-1}(x,y)\right]\right|\,dx\,dy.$

D. Both **A** and **B**

*__E.__ Both **A** and **C**

Question 5 Consider the integral

$$\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dx\, dy\, dz$$

over the unit ball B given by $x^2 + y^2 + z^2 \leq 1$.

A. It may be integrated by using the change of variables $\Phi(\rho, \theta, \phi) = (\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi))$.

B. It may be written in the form $\displaystyle\int_{\rho=0}^{1} \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} e^{\rho^3} \rho^2 \sin(\phi)\, d\phi\, d\theta\, d\rho$.

C. It is an easy integral to evaluate after changing to spherical coordinates.

D. It cannot be evaluated analytically since $e^{(x^2+y^2+z^2)^{3/2}}$ does not have an elementary antiderivative.

*__E.__ **A**, **B**, and **C**