Math 20E

August 17, 2017

Question 1 Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

The lengths of the corresponding curves are

*__A.__ the same since the curves defined by the paths are the same.

B. of opposite sign since the paths traverse the curves in opposite directions.

C. cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

D. both **B** and **C**.

E. none of the above, the two lengths are different.

Question 2 Given a C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. For every $t \in [a, b]$, we can conclude that

A. $\boldsymbol{\tau}_1(t) := \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is a unit tangent vector to $\mathbf{c}$ at $\mathbf{c}(t)$.

B. $\boldsymbol{\tau}_2(t) := -\frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is a unit tangent vector to $\mathbf{c}$ at $\mathbf{c}(t)$.

C. **A**, provided $\mathbf{c}$ is a regular path.

***D.** both **A** and **B**, provided $\mathbf{c}$ is a regular path.

E. none of the above, $\mathbf{c}$ has no unit tangent vectors.

Question 3 Given $\mathbf{F}:\mathbb{R}^n \to \mathbb{R}^n$ a continuous vector field in $\mathbb{R}^n$, with $\mathbf{c}:[a,b]\to\mathbb{R}^n$ a regular path in $\mathbb{R}^n$. Then, $\boldsymbol{\tau}(t)=\frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is defined at each point $\mathbf{c}(t)$ along the path $\mathbf{c}$ and it follows that

A. $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}\left(\mathbf{c}\left(t\right)\right)\cdot\mathbf{c}'(t)\,dt$

B. $\displaystyle\int_{\mathbf{c}} \left(\mathbf{F}\cdot\boldsymbol{\tau}\right)\,ds = \int_a^b \left(\mathbf{F}\left(\mathbf{c}\left(t\right)\right)\cdot\frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}\right)\left\|\mathbf{c}'(t)\right\|\,dt$

C. $\displaystyle\int_a^b \left(\mathbf{F}\left(\mathbf{c}\left(t\right)\right)\cdot\frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}\right)\left\|\mathbf{c}'(t)\right\|\,dt = \int_a^b \mathbf{F}\left(\mathbf{c}\left(t\right)\right)\cdot\mathbf{c}'(t)\,dt$

D. $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_{\mathbf{c}} \mathbf{F}\cdot\boldsymbol{\tau}\,ds$

*__E.__ All of the above.

Question 4 Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

Let $\mathbf{F}(x, y, z)$ be any C^1 vector field. The value of the line integrals $\int_{\mathbf{c}_1} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{c}_2} \mathbf{F} \cdot d\mathbf{s}$ are

A. the same since the curves defined by the paths are the same.

***B.** of opposite sign since the paths traverse the curves in opposite directions.

C. cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

D. both **B** and **C**

E. none of the above

Question 5 Given $f : \mathbb{R}^n \to \mathbb{R}$ a C^1 scalar-valued function and $\mathbf{c} : [a, b] \to \mathbb{R}^n$ a simple C^1 path,

A. $\mathbf{D}f(t) = \nabla f\left(\mathbf{c}\left(t\right)\right) \cdot \mathbf{c}'(t)$ by the chain rule.

B. $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = \int_a^b \nabla f\left(\mathbf{c}\left(t\right)\right) \cdot \mathbf{c}'(t)\, dt.$

C. The value of $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s}$ is independent of the path $\mathbf{c}$.

D. $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = f\left(\mathbf{c}\left(b\right)\right) - f\left(\mathbf{c}\left(a\right)\right)$, which depends only on the value of f at the endpoints $\mathbf{c}(a)$ and $\mathbf{c}(b)$.

*__E.__ All of the above.