Math 20E

August 21, 2017

Question 1 Two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ are orthogonal (or perpendicular or normal) if and only if $\mathbf{v} \cdot \mathbf{w} = \mathbf{0}$. Thus,

A. the angle between two nonzero orthogonal vectors is $\frac{\pi}{2}$

B. the zero vector is orthogonal to every vector.

C. the zero vector is normal to every plane.

D. the zero vector is the only vector orthogonal to itself.

***E.** All of the above.

Question 2 Given a C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is *regular* at t_0 if and only if

A. the image curve $\mathbf{c}([a, b])$ has a tangent line at $\mathbf{c}(t_0)$.

B. $\mathbf{c}'(t_0)$ is a unit vector.

C. $\mathbf{c}'(t_0)$ is tangent to $\mathbf{c}$ at $\mathbf{c}(t_0)$.

D. $\mathbf{c}'(t_0) \neq \mathbf{0}$.

*__E.__ both **C** and **D**; they are equivalent.

Question 3 Given a parametrized surface $\Phi : D \to \mathbb{R}^3$. Φ is *regular* at $\Phi(u_0, v_0)$ if and only if

A. the vector $\mathbf{T}_u \times \mathbf{T}_v$ is normal to the surface $S = \Phi(D)$ at (u_0, v_0).

B. $\mathbf{T}_u \times \mathbf{T}_v$ at (u_0, v_0) is a unit vector.

C. the surface $S = \Phi(D)$ has a tangent plane at $\Phi(u_0, v_0)$.

*__D.__ the vector $\mathbf{T}_u \times \mathbf{T}_v$ is not zero at (u_0, v_0).

E. both **C** and **D**.

Question 4 The surface $x^2 + y^2 = z^2$ for $0 \le z \le 1$ is parametrized by $\Phi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 2\pi] \times [0, 1]$ and $\Phi(u, v) = v\,(\cos(u),\ \sin(u),\ 1)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = v\,(\cos(u),\ \sin(u),\ -1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

D. **A**, **B** and **C**

***E.** none of the above

Question 5 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u,\ v,\ 1 - u^2 - v^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (2u,\ 2v,\ 1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

*__D.__ **A**, **B** and **C**

E. none of the above

Question 6 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(u, v) = (u\cos(v),\ u\sin(v),\ 1 - u^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = u\,(2u\cos(v),\ 2u\sin(v),\ 1)$ and

A. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

B. The parametrized surface Ψ is regular at every point of S.

***C.** The surface $S = \Psi(R)$ has a tangent plane at every point of S.

D. **A**, **B** and **C**

E. none of the above