Math 20E

August 22, 2017

Question 1 Given a continuous vector field $\mathbf{F}$ on $\mathbb{R}^n$ and a regular C^1 path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. Then,

A. The unit tangent vector $\boldsymbol{\tau}(t) = \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ is defined at each point along the path $\mathbf{c}$.

B. The line integral of $\mathbf{F}$ along $\mathbf{c}$ is given by
$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\, dt.$$

C. The path integral of the tangential component of $\mathbf{F}$ along $\mathbf{c}$ is given by
$$\int_{\mathbf{c}} (\mathbf{F} \cdot \boldsymbol{\tau})\ ds = \int_a^b \left(\mathbf{F}(\mathbf{c}(t)) \cdot \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|} \right) \|\mathbf{c}'(t)\|\ dt.$$

D. $\displaystyle\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \int_{\mathbf{c}} (\mathbf{F} \cdot \boldsymbol{\tau})\ ds.$

***E.** All of the above.

Question 2 The surface $x^2 + y^2 = z^2$ for $0 \leq z \leq 1$ is parametrized by $\Phi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 2\pi] \times [0, 1]$ and $\Phi(u, v) = v\,(\cos(u),\ \sin(u),\ 1)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = v\,(\cos(u),\ \sin(u),\ -1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

D. All of the above.

*__E.__ None of the above.

Question 3 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u,\ v,\ 1 - u^2 - v^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (2u,\ 2v,\ 1)$ and

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

***D.** All of the above.

E. None of the above.

Question 4 The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0,1] \times [0,2\pi]$ and $\Psi(u,v) = (u\cos(v),\ u\sin(v),\ 1 - u^2)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = u\,(2u\cos(v),\ 2u\sin(v),\ 1)$ and

A. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

B. The parametrized surface Ψ is regular at every point of S.

*__C.__ The surface $S = \Psi(R)$ has a tangent plane at every point of S.

D. All of the above.

E. None of the above.

Question 5 The surface S given by $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$.

The surface S is given by $x^2 + y^2 + z = 1$ for $z \geq 0$ also parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(r, \theta) = (r \cos(\theta), r \sin(\theta), 1 - r^2)$.

We can conclude that

A. Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

C. $\Psi = \Phi \circ T$, where $T : R \to D$ is the polar coordinate transformation.

***D. A** and **C**.

E. A, **B** and **C**.

Question 6 Let $S \subset \mathbb{R}^3$ be a surface with a regular C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u, v) = (x\,(u, v)\,,\, y\,(u, v))\,,\, z\,(u, v))\,.$$

Then,

A. $\mathbf{T}_u = \frac{\partial \Phi}{\partial u}$ and $\mathbf{T}_v = \frac{\partial \Phi}{\partial v}$ are vectors tangent to S.

B. $\mathbf{T}_u \times \mathbf{T}_v$ is a vector normal to S.

C. The area of $\Phi([u, u + \Delta u] \times [v, v + \Delta v])$ on S is approximately $||\mathbf{T}_u(u, v) \times \mathbf{T}_u(u, v)||\, \Delta u\, \Delta v$.

D. **A** and **B**.

*__E.__ **A**, **B** and **C**.