Math 20E

August 24, 2017

Question 1 The surface $z = \sqrt{x^2 + y^2}$ for $0 \leq z \leq 1$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where D is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = \left(u, v, \sqrt{u^2 + v^2}\right)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = \left(-\frac{u}{\sqrt{u^2+v^2}}, -\frac{v}{\sqrt{u^2+v^2}}, 1\right)$ and

*__A.__ Φ is a one-to-one mapping of D onto $S = \Phi(D)$.

B. The parametrized surface Φ is regular at every point of S.

C. The surface $S = \Phi(D)$ has a tangent plane at every point of S.

D. **A**, **B** and **C**

E. none of the above

Question 2 The surface $z = \sqrt{x^2 + y^2}$ for $0 \le z \le 1$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where R is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(u, v) = (u\cos(v), u\sin(v), u)$. Then, $\mathbf{T}_u \times \mathbf{T}_v = (-u\cos(v), -u\sin(v), u)$ and

A. Ψ is a one-to-one mapping of R onto $S = \Psi(R)$.

B. The parametrized surface Ψ is regular at every point of S.

C. The surface $S = \Psi(R)$ has a tangent plane at every point of S.

D. **A**, **B** and **C**.

***E.** None of the above.

Question 3 Given a surface S with a C^1 regular parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))$$

Let $f(x,y,z)$ be a continuous function defined on S. Then,

A. $\displaystyle\iint_S f\, dS = \iint_D f\,(\Phi\,(u,v))\,||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv.$

B. $\displaystyle\iint_S f\, dS = \iint_D f\,(\Phi\,(u,v))\,||\mathbf{T}_v \times \mathbf{T}_u||\, dv\, du.$

C. The average value of f on S is $\displaystyle\frac{1}{A(S)}\iint_S f\, dS$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv.$$

D. **A** and **B**: they are the same.

***E.** All of the above.

Question 4 Given a surface with two distinct C^1 parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, then

*A. $\iint_\Phi f\,dS = \iint_\Psi f\,dS$.

B. $\iint_\Phi f\,dS < \iint_\Psi f\,dS$ when $||\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}|| < ||\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}||$.

C. $\iint_\Phi f\,dS = -\iint_\Psi f\,dS$ when $\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} = -\left(\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\right)$.

D. **B** and **C**.

E. None of the above.

Question 5 Given a surface S with C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = \left(x\left(u,v\right),\, y\left(u,v\right)\right),\, z\left(u,v\right)\right),$$

and given $\mathbf{F}(x,y,z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_u \times \mathbf{T}_v\right)\, du\, dv.$

B. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = -\iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_v \times \mathbf{T}_u\right)\, dv\, du.$

C. The average value of $\mathbf{F}$ on S is $\displaystyle\frac{1}{A(S)}\iint_S \mathbf{F}\cdot d\mathbf{S}$,

where $\displaystyle A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\, du\, dv.$

*__D.__ **A** and **B**.

E. **A**, **B** and **C**.