Math 20E

August 25, 2017

Question 1 Set $D = \left\{(u,v) \mid u^2+v^2 \leq 1\right\}$ and $R = [-\pi,\pi] \times [0,\pi/2]$. $\Phi : D \to S$ and $\Psi : R \to S$ with

$$\Phi(u,v) = \left(u,\ v,\ \sqrt{1-u^2-v^2}\right)$$
$$\Psi(\theta,\phi) = (\cos(\theta)\cos(\phi),\ \sin(\theta)\cos(\phi),\ \sin(\phi))$$

are both parametrizations of the upper unit hemisphere $S : x^2+y^2+z^2 = 1$, with $z \geq 0$.

*****A.** $\displaystyle\iint_\Phi f\,dS = \iint_\Psi f\,dS$ for every continuous function f.

B. $\displaystyle\iint_\Psi f\,dS$ is not defined because Ψ is not a one-to-one transformation.

C. $\displaystyle\iint_\Psi f\,dS$ is not defined because Ψ is not regular at every point of S.

D. **B** and **C**.

E. None of the above.

Question 2 Given $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ a continuous vector field in $\mathbb{R}^n$, with $\Phi : D \to S$ a regular parametrized surface in $\mathbb{R}^3$. Then, $\mathbf{n} = \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}$ is defined at each point $\Phi(u, v)$ on the surface S and it follows that

A. $\displaystyle \iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\Phi(u,v)) \cdot (\mathbf{T}_u \times \mathbf{T}_v)\, du\, dv.$

B. $\displaystyle \iint_\Phi (\mathbf{F} \cdot \mathbf{n})\, dS =$
$$\iint_D \left(\mathbf{F}(\Phi(u,v)) \cdot \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|} \right) \|\mathbf{T}_u \times \mathbf{T}_v\|\, du\, dv.$$

C. $\displaystyle \iint_D \mathbf{F}(\Phi(u,v)) \cdot (\mathbf{T}_u \times \mathbf{T}_v)\, du\, dv =$
$$\iint_D \left(\mathbf{F}(\Phi(u,v)) \cdot \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|} \right) \|\mathbf{T}_u \times \mathbf{T}_v\|\, du\, dv.$$

D. $\displaystyle \iint_\Phi \mathbf{F} \cdot d\mathbf{S} = \iint_\Phi (\mathbf{F} \cdot \mathbf{n})\, dS.$

*__E.__ All of the above.

Question 3 Given a surface with two distinct C^1 parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, and given $f(x,y,z)$ a continuous function defined on S. Then,

***A.** $\displaystyle\iint_\Phi f\,dS = \iint_\Psi f\,dS.$

B. $\displaystyle\iint_\Phi f\,dS < \iint_\Psi f\,dS$ when $||\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}|| < ||\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}||$.

C. $\displaystyle\iint_\Phi f\,dS = -\iint_\Psi f\,dS$ when $\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v} = -\left(\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}\right)$.

D. **B** and **C**.

E. None of the above.

Question 4 Given a surface S with C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))\,,$$

and given $\mathbf{F}(x,y,z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_u \times \mathbf{T}_v\right)\,du\,dv.$

B. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = -\iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_v \times \mathbf{T}_u\right)\,dv\,du.$

C. The average value of $\mathbf{F}$ on S is $\displaystyle\frac{1}{A(S)}\iint_S \mathbf{F}\cdot d\mathbf{S}$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv.$$

***D. A** and **B**.

E. A, **B** and **C**.

Question 5 Given a surface with two distinct C^1 parametrizations $\Phi : D \to S$ and $\Psi : D \to S$, and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S. Then,

*__A.__ $\displaystyle\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$.

B. $\displaystyle\iint_{\Phi} \mathbf{F}\cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F}\cdot d\mathbf{S}$ when $\displaystyle\frac{\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}}{||\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}||} = \frac{\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}}{||\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}||}$.

C. $\displaystyle\iint_{\Phi} \mathbf{F}\cdot d\mathbf{S} = -\iint_{\Psi} \mathbf{F}\cdot d\mathbf{S}$ when $\displaystyle\frac{\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}}{||\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}||} = -\frac{\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}}{||\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}||}$.

D. $\displaystyle\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} < \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $||\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}|| < ||\frac{\partial\Psi}{\partial u} \times \frac{\partial\Psi}{\partial v}||$.

E. **B** and **C**.