Math 20E

August 28, 2017

Question 1 S a surface with C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\, y\,(u,v))\,,\, z\,(u,v))\,.$$

Let $f(x,y,z)$ be a continuous function defined on S. Then,

A. $\displaystyle\iint_S f\,dS = \iint_D f\left(\Phi\left(u,v\right)\right)||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv.$

B. $\displaystyle\iint_S f\,dS = \iint_D f\left(\Phi\left(u,v\right)\right)||\mathbf{T}_v \times \mathbf{T}_u||\,dv\,du.$

C. The average value of f on S is $\displaystyle\frac{1}{A(S)}\iint_S f\,dS$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv.$$

D. **A** and **B**: they are the same.

***E.** **A**, **B** and **C**.

Question 2 S a surface with C^1 parametrization $\Phi : D \to S$, where

$$\Phi(u,v) = (x\,(u,v)\,,\,y\,(u,v))\,,\,z\,(u,v))\,,$$

and $\mathbf{F}(x,y,z)$ a continuous vector field defined on S. Then,

A. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_u \times \mathbf{T}_v\right)\,du\,dv.$

B. $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = -\iint_D \mathbf{F}\left(\Phi\left(u,v\right)\right)\cdot\left(\mathbf{T}_v \times \mathbf{T}_u\right)\,dv\,du.$

C. The average value of $\mathbf{F}$ on S is $\displaystyle\frac{1}{A(S)}\iint_S \mathbf{F}\cdot d\mathbf{S}$, where

$$A(S) = \iint_D ||\mathbf{T}_u \times \mathbf{T}_v||\,du\,dv.$$

*__D.__ **A** and **B**.

E. **A**, **B** and **C**.

Question 3 Given S a surface with regular C^1 parametrization $\Phi : D \to S$, and $\mathbf{F}(x, y, z)$ a continuous vector field defined on S. Then,

A. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n})\ dS$, where $\mathbf{n}$ is the unit normal vector at each point of S given by orientation of S.

B. The unit normal vector $\mathbf{n}$ at each point of the parametrized surface Φ is given by $\mathbf{n} = \dfrac{\mathbf{T}_u \times \mathbf{T}_v}{||\mathbf{T}_u \times \mathbf{T}_v||}$.

C. The average value of the normal component of $\mathbf{F}$ on S is $\dfrac{1}{A(S)} \iint_S \mathbf{F} \cdot d\mathbf{S}$, where $A(S)$ is the area of the surface S.

D. **A** and **B**.

***E.** **A**, **B** and **C**.

Question 4 Given a simple domain D with C^1 boundary ∂D, the area of D is given by

A. $A(D) = \iint_D dx\, dy.$

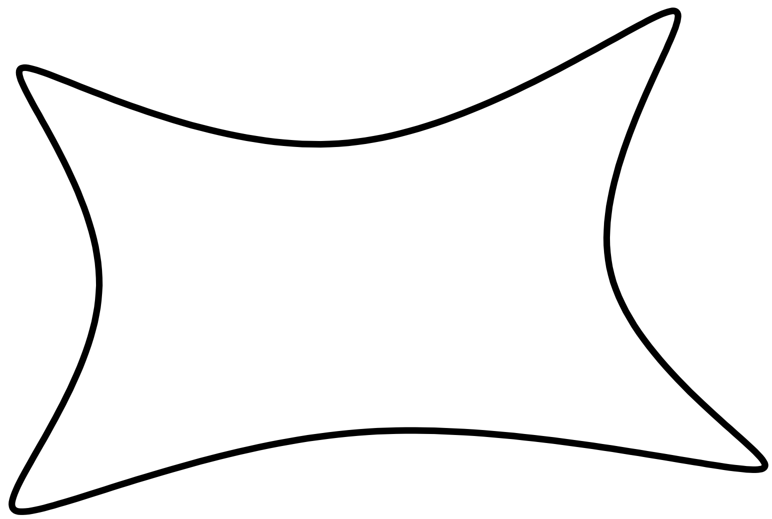
B. $A(D) = -\int_{\partial D} y\, dx.$

C. $A(D) = \frac{1}{2}\int_{\partial D} x\, dy - y\, dx.$

D. **A** and **C**.

***E.** **A**, **B** and **C**.

Question 5 Consider the following domain D:



A. By Green's theorem, the area of D may be computed by evaluating $\frac{1}{2}\int_{\partial D} x\,dy - y\,dx$.

B. Green's theorem cannot be applied on D since D is not a simple region.

C. The area of D could be measured by tracing ∂D with a planimeter.

***D.** **A** and **C**.

E. **B** and **C**.