Math 20E

August 29, 2017

Question 1 Given a simple domain D with C^1 boundary ∂D, the area of D is given by

A. $A(D) = \iint_D dx\, dy.$

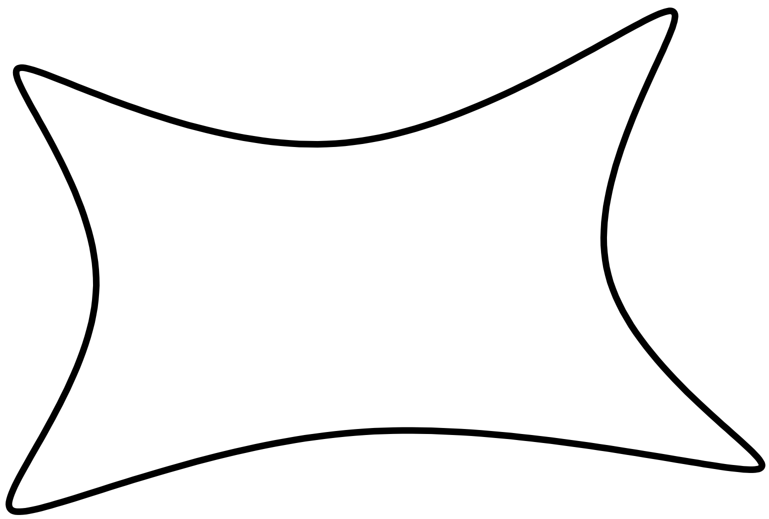
B. $A(D) = -\int_{\partial D} y\, dx.$

C. $A(D) = \frac{1}{2}\int_{\partial D} x\, dy - y\, dx.$

D. **A** and **C**.

*__E.__ **A**, **B** and **C**.

Question 2 Consider the following domain D:



A. By Green's theorem, the area of D may be computed by evaluating $\frac{1}{2}\int_{\partial D} x\,dy - y\,dx$.

B. Green's theorem cannot be applied on D since D is not a simple region.

C. The area of D could be measured by tracing ∂D with a planimeter.

***D.** **A** and **C**.

E. **B** and **C**.

Question 3 Given a surface S with $\Phi : D \to S$ a regular C^1 parametrization of S, and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S. Then,

A. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n})\, dS$, where $\mathbf{n}$ is the unit normal vector at each point of S given by orientation of S.

B. The unit normal vector $\mathbf{n}$ at each point of the parametrized surface Φ is given by $\mathbf{n} = \dfrac{\mathbf{T}_u \times \mathbf{T}_v}{||\mathbf{T}_u \times \mathbf{T}_v||}$.

C. The average value of the normal component of $\mathbf{F}$ on S is $\dfrac{1}{A(S)} \iint_S \mathbf{F} \cdot d\mathbf{S}$, where $A(S)$ is the area of the surface S.

D. **A** and **B**.

***E.** **A**, **B** and **C**.

Question 4 Given an orientable surface S with boundary curve C, and a C_1 vector field $\mathbf{F}$. Then,

A. $\displaystyle\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{s}$.

B. Given a path $\mathbf{c}$ that parametrizes the curve C, $\displaystyle\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \pm \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, depending on the orientation chosen for S.

C. Given a parametrization $\Phi : D \to S$,

$$\iint_{\Phi} \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_{\partial\Phi} \mathbf{F} \cdot d\mathbf{s},$$

where $\partial\Phi$ is the positively oriented boundary curve with respect to the orientation of Φ.

*__D.__ **A** and **C**.

E. **B** and **C**.

Question 5 Given a one-to-one parametrized surface $\Phi : D \to S \subset \mathbb{R}^3$. Then,

A. $\Phi : \partial D \to S$ parametrizes the boundary curve $\partial\Phi$ of the parametrized surface Φ.

B. The orientation of $\partial\Phi$ is determined by the orientation of ∂D.

C. Stokes' Theorem on Φ follows from Green's Theorem on D.

*__D.__ All of the above.

E. None of the above. George Stokes and George Green never met.