Math 20E

August 31, 2017

Question 1 Given a one-to-one parametrized surface $\Phi : D \to S \subset \mathbb{R}^3$. Then,

A. $\Phi : \partial D \to S$ parametrizes the boundary curve $\partial\Phi$ of the parametrized surface Φ.

B. The orientation of $\partial\Phi$ is determined by the orientation of ∂D.

C. Stokes' Theorem on Φ follows from Green's Theorem on D.

***D.** All of the above.

E. None of the above. George Stokes and George Green never met.

Question 2 Given a scalar-valued function $f(x, y, z)$ defined on a parametrized surface Φ whose image is an oriented surface $S \in \mathbb{R}^3$. Then,

A. $\iint_\Phi f\, dS$ is called the integral of f over the parametrized surface Φ.

B. $\iint_\Phi f\, dS$ is called the integral of f over the surface S since it is independent of the choice of the parametrized surface Φ.

C. $\iint_\Phi f\, dS$ is called the flux of f across the surface S.

*__D.__ **A** and **B**.

E. All of the above.

Question 3 Given a vector-valued function $\mathbf{F}(x, y, z)$ defined on a parametrized surface Φ whose image is an oriented surface $S \in \mathbb{R}^3$. Then,

A. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S}$ is called the integral of $\mathbf{F}$ over the parametrized surface Φ.

B. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S}$ is the integral of the normal component of $\mathbf{F}$ over the parametrized surface Φ.

C. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S}$ is called the flux of $\mathbf{F}$ across the surface S.

D. **A** and **B**.

***E.** All of the above.

Question 4 Given a C^1 vector field $\mathbf{F}$ in $\mathbb{R}^3$. Let S be an oriented surface with positively-oriented boundary curve ∂S. Then,

A. $\displaystyle\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$ is called the circulation of $\mathbf{F}$ around ∂S.

B. $\displaystyle\iint_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S}$ is called the flux of the curl of $\mathbf{F}$ across S.

C. the circulation of $\mathbf{F}$ around ∂S is equal to the flux of the curl of $\mathbf{F}$ across S.

***D.** all of the above.

E. none of the above.

Question 5 Suppose $\mathbf{F}$ is a C^1 vector field on the unit sphere S in $\mathbb{R}^3$. Then, $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. is 0.

B. is most easily computed by parametrizing S using latitude-longitude coordinates.

C. is most easily computed by applying Stokes' theorem and computing $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

D. cannot be computed using Stokes' theorem because the sphere S has no boundary curve ∂S.

***E.** **A** and **C**: the line integral in **C** is 0 because the boundary curve ∂S is empty.

Question 6 Suppose $\mathbf{F}$ is a C^1 vector field on $\mathbb{R}^3$. Let H be the unit hemisphere given by $x^2+y^2+z^2=1$ with $z \geq 0$, let D be the unit disk given by $z=0$ with $x^2+y^2 \leq 1$, and let $S = H \cup D$. Then,

A. $\partial H = \partial D$, including orientation when H and D are both oriented with the upward-pointing unit normal vector.

B. $\displaystyle\iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}.$

C. $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0.$

D. $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_H (\nabla \times \mathbf{F}) \cdot d\mathbf{S} + \iint_D (\nabla \times \mathbf{F}) \cdot d\mathbf{S}.$

*__E.__ **A**, **B** and **C**