Math 20E
Midterm Exam 2
March 2, 2012
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Version A

Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write the *Version* of your exam on the front of your Blue Book.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
6. **Read each question carefully, and answer each question completely.**
7. **Show all of your work; no credit will be given for unsupported answers.**

1. A helical wire follows the path $\mathbf{c}(t) = (3\cos(t), 3\sin(t), 4t)$ for $0 \le t \le 5\pi$. Its mass density λ (mass per unit length) is given by $\lambda(x, y, z) = 2z$. Find the mass of the wire.

2. Let S be the parabolic surface given by $z = 9 - x^2 - y^2$ for $x^2 + y^2 \le 9$.

   (a) Find a parametrization $\Phi : D \to S$. Be sure to specify the domain D.

   (b) Use your parametrization to find a normal vector to S at the point $(1, 1, 7)$.

3. The conical surface S given by $x^2 + y^2 = (3 - z)^2$ with $z \ge 0$ can be parametrized by

$$\Phi : [0, 2] \times [0, 2\pi] \longrightarrow S \subset \mathbb{R}^3$$
$$\Phi(r, \theta) = (r\cos(\theta), r\sin(\theta), 3 - r)$$

   Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}$ is the radial vector field $\mathbf{F}(x, y, z) = (x, y, z)$.

4. Given $a > c > 0$, the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{c^2} = 1$ is an equation for an ellipse centered at the origin with semimajor axis a and semiminor axis c.

   (a) Verify that $\mathbf{c}(t) = (a\cos(t), c\sin(t))$ for $0 \le t \le 2\pi$ is a parametrization for the ellipse.

   (b) Use Green's theorem to compute the area enclosed by the ellipse.