

Math 20E
Formula Sheet

Vector Identities

1. $\nabla(f + g) = \nabla f + \nabla g$
2. $\nabla(cf) = c \nabla f$
3. $\nabla(fg) = f \nabla g + g \nabla f$
4. $\nabla\left(\frac{f}{g}\right) = \frac{g \nabla f - f \nabla g}{g^2}$, at points \mathbf{x} where $g(\mathbf{x}) \neq 0$
5. $\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$
6. $\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$
7. $\nabla \cdot (f\mathbf{F}) = f \nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla f$
8. $\nabla \times (f\mathbf{F}) = f \nabla \times \mathbf{F} + \nabla f \times \mathbf{F}$
9. $\nabla \cdot (\nabla \times \mathbf{F}) = 0$
10. $\nabla \times \nabla f = \mathbf{0}$
11. $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$

Selected Integrals

1. $\int \tan(x) dx = \log |\sec(x)|$
2. $\int \sec(x) dx = \log |\sec(x) + \tan(x)|$
3. $\int \arcsin\left(\frac{x}{a}\right) dx = x \arcsin\left(\frac{x}{a}\right) + \sqrt{x^2 + a^2}$
4. $\int \arctan\left(\frac{x}{a}\right) dx = x \arctan\left(\frac{x}{a}\right) - \frac{a}{2} \log(a^2 + x^2)$
5. $\int \sin^2(x) dx = \frac{1}{2}(x - \sin(x)\cos(x))$
6. $\int \cos^2(x) dx = \frac{1}{2}(x + \sin(x)\cos(x))$
7. $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log(x + \sqrt{a^2 + x^2})$
8. $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right)$
9. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right)$
10. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right)$
11. $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \log(x + \sqrt{x^2 \pm a^2})$
12. $\int \frac{1}{(a^2 - x^2)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
13. $\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log(x + \sqrt{x^2 \pm a^2})$
14. $\int (a^2 - x^2)^{3/2} dx = \frac{x}{8}(5a^2 - 2x^2)\sqrt{a^2 - x^2} + \frac{3a^4}{8} \arcsin\left(\frac{x}{a}\right)$

Path and Line Integrals over a Parameterized Curve (Path) $\mathbf{c}(t)$

1. Path integral of a scalar function f : $\int_{\mathbf{c}} f ds = \int_a^b f(\mathbf{c}(t)) \|\mathbf{c}'(t)\| dt$
2. Line integral of a vector field \mathbf{F} : $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) dt$

Surface Integrals over a Parameterized Surface $\Phi(u, v)$

1. Integral of a scalar function f : $\iint_S f dS = \iint_D f(\Phi(u, v)) \|\mathbf{T}_u \times \mathbf{T}_v\| du dv$
2. Integral of a vector field \mathbf{F} : $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\Phi(u, v)) \cdot (\mathbf{T}_u \times \mathbf{T}_v) du dv$

Green's Theorem: $\int_{\partial D} P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx dy$

Special Case: $\frac{1}{2} \int_{\partial D} x dy - y dx = \iint_D dx dy$

Stoke's Theorem: $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$

Gauss' Theorem: $\iiint_W (\nabla \cdot \mathbf{F}) dV = \iint_{\partial W} \mathbf{F} \cdot d\mathbf{S}$