Topology 190, Fall 2001. Midterm

A. Calculate the numbers of three-colourings of the two two-component links shown below. Are these links equivalent? (15 marks)

B. State Reidemeister's theorem on equivalence of (unoriented) links. (10 marks)

C. Say whether each of the following statements is true or false. (You don't have to give any explanation, or write anything other than "true" or "false".) (15 marks)

1. If K_1, K_2 are knots with $\tau_3(K_1) = \tau_3(K_2)$ then K_1 is equivalent to K_2.
2. If K_1, K_2 are equivalent knots then $\tau_3(K_1) = \tau_3(K_2)$.
3. If K_1, K_2 are knots with $\tau_3(K_1) = \tau_3(K_2)$, then $\tau_5(K_1) \neq \tau_5(K_2)$.
4. If an oriented link L has $\mathrm{Lk}(L) = 0$, then L is equivalent to an unlink.
5. A knot K for which $\tau_7(K) = 7$ must be an unknot.
6. There is no knot K with $\tau_3(K) = 6$.
7. For any oriented knot K, $\tau_5(rK) = \tau_5(K)$. (Here, rK denotes the orientation-reversed version of K.)
8. For any knot K, $\tau_5(\bar{K}) = -\tau_5(K)$. (Here, $\bar{K}$ denotes the mirror-image of K.)
9. For any oriented link L, $\mathrm{Lk}(rL) = -\mathrm{Lk}(L)$. (Here, rL denotes the link obtained by reversing the orientations of all the components of L.)
10. The function f defined on knot diagrams by setting $f(D) =$ (number of crossings of D) defines an invariant of knots.
11. For any n, the number of knots with crossing number at most n is finite.
12. For any n, the number of knots with unknotting number at most n is finite.
13. If L is an oriented link with n components, then $|\mathrm{Lk}(L)| \leq n$.
14. If L is an oriented link which has a diagram with c crossings, then $|\mathrm{Lk}(L)| \leq \frac{1}{2}c$.
15. If L is a link with n components then $\tau_2(L) = 2^n$.